UNIVERSITY OF SWAZILAND

DEPARTMENT OF COMPUTER SCIENCE

CSC203 — DESCRETE MATHEMATICS

FINAL EXAMINATION

DECEMBER 2018

Instructions

1. The time allowed is **THREE (3) HOURS**.
2. Read all the questions before you start answering any question.
3. There are six (6) qustions. Answer **any four(4)** questions. Each question has 25 marks. Maximum mark is 100.
4. Use correct notation and show all your work on the answer script.

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Answer **any four (4)** questions.

Question 1 [25]

a **[4]** With the aid of suitable examples, define the following terms used in logic:

  a Clause

  b Predicate

b **[3]** Given the conditional statement , "For Sipho to get a good job, it is sufficient for him to learn discrete mathematics", write down the statements for contrapositive, inverse and converse.

c **[3]** Evaluate this expression $1100 \wedge (01011 \vee 11011)$.

d **[4]** What is the value of x after the statement

$$\textbf{if}(x+1=3) \text{ OR } (2x+2=3) \textbf{ then } x := x+1$$

encountered in a computer program, if $x = 1$ before the statement is reached?

e **[5]** Use truth tables to show that $(p \vee q) \rightarrow r$ is equivalent to $(p \rightarrow r) \wedge (q \rightarrow r)$.

f **[4]** Show that $(p \wedge q) \rightarrow r$ is a tautology.

g **[2]** Let $P(x)$ be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English.

  a $\exists x P(x)$

  b $\forall x P(x)$

Question 2 [25]

a **[5]** Draw a Venn diagram for the following sets of numbers: $\mathbb{C}, \mathbb{Z}, \mathbb{Q}, \mathbb{N}, \mathbb{R}$

b **[2]** Differentiate between an open interval and a closed interval.

c **[3]** Let A be the set $\{x, y, z\}$ and B be the set $\{x, y\}$.

  i Is A a subset of B?

  ii What is $A \times B$

  iii What is the power set of B?

d **[6]** Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

e **[2]** Why is $f(x) = 1/x$ not a function from $\mathbb{R}$ to $\mathbb{R}$?

f [4] With the aid of suitable examples, differentiate between a total function and a partial function.

g [3]Find these terms of the sequence an $\{a_n\}$ where $a_n = 2^n + 1$

i a_0

ii a_4

Question 3 [25]

a [5] List five (5) characteristics of an algorithm.

b [4 + 4 +1] Consider an algorithm for finding the smallest integer in a list of n integers.

i Describe the algorithm using English.

ii Express this algorithm in pseudocode

iii How many comparisons does the algorithm use?

c [4] Explain the halting problem.

d [4] What is the order $O(f(x))$ of the following functions

i $f(x) = 17x + 11$

ii $f(x) = 2^x$

iii $f(x) = (x^2 + 1)/(x + 1)$

e [3] Show that $(n \log n + n^2)^3$ is $O(n^6)$.

Question 4 [25]

a [4] Does 17 divide each of these numbers?

i 68

ii 357

b [6] Suppose that a and b are integers, $a \equiv 4(\mod 13)$, and $b \equiv 9(\mod 13)$. Find the integer c with $0 \leq c \leq 12$ such that

i $c \equiv 9a(\mod 13)$

ii $c \equiv a + b(\mod 13)$

iii [8] Find the octal and hexadecimal expansions of $(11111010111100)_2$ and the binary expansions of $(765)_8$ and $(A8D)_{16}$.

iv [4] What is the greatest common divisor of 17 and 22?

v [4] Find the prime factorization of 126 and 111.

Question 5 [25]

a **[6]** Explain the basic components of (i) mathematical induction and (ii) recursion.

b **[8]** Use mathematical induction to show that

i $1 + 2 + 2^2 + ... + 2^n = 2^{n+1} - 1$

ii $1^2 + 3^2 + 5^2 + ... + (2n+1)^2 = (n+1)(2n+1)(2n+3)/3$

for all positive integers n.

c **[8]** Write down pseudocode that calculate the factorial of an integer n, that is $n!$, using (i) a recursion and (ii) iteration.

d **[3]** Given $f(n+1) = 2^{f(n)}$. Find $f(3)$, if $f(n)$ is defined recursively by $f(0) = 1$ and for n = 0, 1, 2, ...

Question 6 [25]

a **[4]** Briefly, explain the two basic counting principles.

b **[6]** There are 18 mathematics majors and 325 computer science majors at a college.

i In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?

ii In how many ways can one representative be picked who is either a mathematics major or a computer science major?

c **[4]** Show that if there are 30 students in a class, then at least two have last names that begin with the same letter.

d **[3]** How many different permutations are there of the set {a, b, c, d, e, f, g}?

e **[6]** A coin is flipped eight times where each flip comes up either heads or tails. How many possible outcomes

i are there in total?

ii contain exactly three heads?

iii contain the same number of heads and tails?

f **[3]** Find the expansion of $(x + y)^6$.