UNIVERSITY OF SWAZILAND

DEPARTMENT OF COMPUTER SCIENCE

CS211 / CSC211 — THEORY OF COMPUTATION

FINAL EXAMINATION

DECEMBER 2018

Instructions

1. The time allowed is **THREE (3) HOURS**.
2. Read all the questions in **Section A** and **Section B** before you start answering any question.
3. Answer **all** questions in Section A. Answer **any two** questions of Section B. Maximum mark is 100.
4. Use correct notation and show all your work on the answer script.

DO NOT OPEN THIS PAPER UNTIL YOU ARE INSTRUCTED TO DO SO BY THE INVIGILATOR

Section A

Answer **all two** questions in this section.

Question 1 [25]

a [**3**] Let A be the set $\{x, y, z\}$ and B be the set $\{x, y\}$.

i Is A a subset of B?

ii What is $A \times B$

iii What is the power set of B?

b [**4**] Define a grammar for the set of integer numbers in C++.

c [**2**] With an aid of a suitable example, explain the role of a trap state in a DFA.

d [**8+8**] Find (i) the grammars and (ii) the DFAs for the following languages on $\Sigma = \{a, b\}$.

I $L = \{w : |w| \bmod 3 = 2\}$

II $L = \{a^n b^m : m + n \text{ is odd}\}$

Question 2 [25]

a You are given the following grammar $G = (\{S, A, B\}, \{a, b\}, S, P)$ and P is given by;

$S \rightarrow abA$,
$A \rightarrow baB$,
$B \rightarrow aA|bb$

i [**7**] Construct an NFA that accepts the language generated by the grammar above.

ii [**8**] Convert the NFA into a DFA.

b [**10**] Design a finite state automaton that simulates the operations of a microwave oven.

The following states of the microwave oven may be included.

- ON - when the oven is on, lights are on and every key is ready for engagement
- OFF - when the oven is off, electric circuit is switched off.
- IDLE - when the oven is in a state of readyness to microwave heat
- HEAT - when the oven is microwave heating

A finite state automaton has actions which allows them to transit from stare to state, for example a DFA/NFA has a 'read "a" ' action.
The following actions may be included in your finite state machine for a microwave oven.

- Press 'On' - to press a key 'On' to power on the microwave oven
- Press 'Off' - to press a key 'Off' to power off the microwave oven
- Prepare settings - this action is a set (or combination) of actions for preparation ot microwave for example setting the cooking timer and placing the food into the oven
- Stop settings - invoked when the microwave oven is done cooking, for example cooking timer is 0.
- Press "Start" - to press a key 'Start' to instantiate a heating session of a microwave oven
- Press "Stop" - to press a key 'Stop' to terminate a heating session of a microwave oven
- Press "Pause" - to press a key 'Pause' to temporalily halt a heating session of a microwave oven, which may be resumed later
- Open door - an action for physically opening the door of the microwave oven
- Close door - an action for physically closing the door of the microwave oven

NOTE: You must specify (and explain) additional assumptions that you have added into your design of a finite state automaton for a microwave oven. For example, clearly stating your your choosen initial state and final states.

Section B

Answer **any two** questions in this section.

Question 3 [25]

a **[6]** The operation $tail(L)$ is defined as

$$tail(L) = \{y : xy \in L \text{ and } x, y \in \Sigma^*\}$$

Show that the family of regular languages is closed under this operation.

b **[6]** Show that the language, $L = \{a^n b^n c^n\}$, is not regular.

c **[4 + 4 + 4]** Given the context free grammar, as ordered below, remove the following productions listed in (i)-(iii).

$S \rightarrow aA|aBB$
$A \rightarrow aaA|\lambda$
$B \rightarrow bB|bbC$
$C \rightarrow B$

i λ productions.

ii unit productions.

iii useless productions.

Question 4 [25]

a **[10 + 5]** Design a deterministic pushdown automaton **(dpda)** to recognize the language—

$$L = \{w \in a^n b^m, n > m\}$$

Describe the functional steps of your **dpda**. Write instantaneous descriptions for $w_1 = aaabb$ and $w_2 = aaabbb$.

b **[6 + 4]** Design a non deterministic pushdown automaton **(npda)** to recognize the language generated by the grammar in Griebach Normal Form —

$$G = (\{S, Z, B\}, \{a, b\}, S, P)$$

where the set of productions P is —

{
$S \rightarrow aBZ|aB|aZ|a$
$B \rightarrow bB|b$
$Z \rightarrow aBZ|aZ|aB|a$
}

Write instantaneous descriptions of your **npda** for $w = aaab$.

Question 5 [15 + 5 + 5]

Write the design of a Turing Machine to compute:

$$F(x) = 3x + 2$$

Assume x to be a non zero positive integer in unary representation. Clearly write the functional steps of your TM computations. Also write the instantaneous descriptions using the values of x as 11 and 1111 (in unary representation) for your Turing Machine.