UNIVERSITY OF SWAZILAND

DEPARTMENT OF COMPUTER SCIENCE

CS211 / CSC211 — THEORY OF COMPUTATION

RE-SIT EXAMINATION

JULY 2019

Instructions

1. Read all the questions in **Section A** and **Section B** before you start answering any question.
2. Answer all questions in Section A. Answer **any two** questions of Section B. Maximum mark is 100.
3. Use correct notation and show all your work on the answer script.

Section A

Question 1 [25]

a [4] Name the essential features of an automaton.

b [5] Show that $L = \{awa : w \in \{a, b\}^*\}$ is regular.

c. [4] Find grammars for $\Sigma = \{a, b\}$ that generate the sets of all strings with no more than three b's.

d [6 + 6] The following languages are given on $\Sigma = \{0, 1\}$. Assume $w \in \{0, 1\}^+$.

I $L1 = \{w, |w| = 2m, m = 1, 2, 3, \dots\}$

II $L2 = \{w,$ such that 01 is always a substring of $w\}$

III $L3 = \{w,$ such that $|w| \bmod 3 = 0\}$

The following set of words is given —

$$\{\lambda, 0, 1, 01, 001, 0100, 00101, 1111, 1111101, 100000, 0011110, 010101\}$$

i From the above set, write all words belonging to L1, all words belonging to L2 and all words belonging to L3.

ii Write the regular expressions representing L1, L2 and L3.

Question 2 [6 + 6 + 14]

The following non deterministic finite acceptor (nfa) is given: $M = (\{q_0, q_1, q_2\}, \{0, 1\}, q_0, \delta, \{q_0, q_2\})$ where the transitions are given as:

$\delta(q_0, 0) = q_2; \delta(q_0, 1) = \{q_0, q_1\};$
$\delta(q_1, 0) = q_2; \delta(q_1, 1) = \{q_1\};$
$\delta(q_2, 0) = q_1; \delta(q_2, 1) = \{q_0\};$

a Draw the transition digraph of the nfa.

b Compute $\delta^*(q_0, w)$ where $w = 0111, 1000$ and 0101

c Convert the above nfa into an equivalent dfa and write its state transition table.

Section B

Question 3 [25]

a [10]A context free grammar, $G = (\{S\}, \{a, b\}, S, \{S \to aS|aSbS|\lambda\})$.

Using G, write leftmost derivations for $w_1 = aaaab$ and $w_2 = abab$. Taking examples of both w_1 and w_2, show that G is ambiguous by drawing two distinct parse trees for each w_1 and w_2. What is the complexity of G?

b [9 + 6] Assuming n, m and $k \geq 0$. Find Context Free Grammars G_1 and G_2 that generate the following languages.

i $L(G_1) = \{a^{2n}b^ic^{2m} : i = m + n\}$

ii $L(G_1) = \{a^nb^mc^{2k} : m = k\}$

Write leftmost derivations for $w_1 = aab, w_2 = bcc, w_3 = aabbcc$ using G_1 and $w_4 = bbcc, w_5 = aabbcc, w_6 = aabc$ using G_2.

Include production number at each step of your derivation.

Question 4 [25]

a [10 + 5] Design a deterministic pushdown automaton (**dpda**) to recognize the language—

$$L = \{w \in a^nb^m, n > m\}$$

Describe the functional steps of your **dpda**. Write instantaneous descriptions for $w_1 = aaabb$ and $w_2 = aaabbb$.

b [6 + 4] Design a non deterministic pushdown automaton (**npda**) to recognize the language generated by the grammar in Griebach Normal Form —

$$G = (\{S, A, B\}, \{a, b\}, S, P)$$

where the set of productions P is —

$\{$
$S \to aABB|aAA$
$A \to aBB|a$
$B \to bBB|A$
$\}$

Write instantaneous descriptions of your **npda** for $w = aaab$.

Question 5 [15 + 5 + 5]

Write the functional steps of the design of a Turing Machine to compute:

$$F(x) = x \text{ div } 3$$

Assume x to be a non zero positive integer in unary representation. Also write the design and instantaneous descriptions using the values of x as 111 and 11111 (in unary representation) for your Turing Machine.