UNIVERSITY OF ESWATINI

Faculty of Science and Engineering

Department of Computer Science

MAIN EXAMINATION
November 2019

Title of Paper: **INTRODUCTION TO LOGIC**

Course Number: CSC201

Time Allowed: 3 hours Total Marks: **100**

Instructions to candidates:

This question paper consists of FIVE (5) questions.
Answer Question 1 and three others.
Marks are indicated in square brackets.
All questions carry equal marks.

SPECIAL REQUIREMENTS:

NO CALCULATORS ALLOWED

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR

Question 1

(a) Define the following terms as used in logic: [2 marks each]

- (i) Sequential circuit
- (ii) Argument
- (iii) Clause
- (iv) Disjunctive Normal Form

(b) State; [2 marks each]

- (i) The advantage(s) of the predicate logic has over propositional logic.
- (ii) The advantage(s) of quine McCluskey method over the K-map method for simplifying digital logic functions.

(c) Draw the truth table of $P \vee Q \leftrightarrow \neg P \wedge \neg Q \wedge R$. [6]

(d) Give an example of deductive reasoning. [2]

(e) Use consistency analysis to check if the following statements are consistent with each other. For what models are they consistent? [5]

If it is raining then the road get wet.
If the road is wet, it becomes slippery.

Question 2

(a) Explain what a contradiction is and give an example. [2]

(b) Use a truth table to prove that $(P \rightarrow Q) \vDash Q$. [4]

(c) Find the DNF and CNF of $P \rightarrow (Q \vee \neg R)$. [6]

(d) Using laws of logical equivalence, prove the following equivalences.

- (i) $P \rightarrow (\neg Q \vee R) \equiv \neg(P \wedge (Q \wedge \neg R))$ [3]
- (ii) $(\neg P \vee Q) \wedge (\neg P \vee S) \wedge (\neg Q \vee \neg S \vee P) \equiv P \leftrightarrow (Q \wedge S)$ [4]

(e) Given ;

$\neg A \rightarrow B$
$B \rightarrow A$
$A \rightarrow (C \wedge D)$

Prove the proposition $A \wedge C \wedge D$ using resolution. [6]

Question 3

(a) What is meant by sufficiency set in digital logic? [1]

(b) Minimize the following functions using Karnaugh map method.

(i) $f(A,B,C,D) = \sum(0,1,2,3,5,7,8,10,12,13,15)$ [5]

(ii) $f(a,b,c) = ab\cdot\bar{c} + \bar{a}\cdot bc + \bar{a}\cdot\bar{b}\cdot c + \bar{a}\cdot\bar{b}\cdot\bar{c}$ impossible input abc. [5]

(c) Use Quine McCluskey method to minimize the following function. [6]

$$f(a,b,c,d) = \bar{a}\cdot\bar{b}cd + \bar{a}\cdot\bar{b}c\bar{d} + \bar{a}\cdot\bar{b}\cdot\bar{c}d + \bar{a}b\bar{c}d + \bar{a}bc\bar{d} + \bar{a}bcd + abcd + abc\bar{d}$$

(d) Use basic gates to draw the circuit that implements the minimized expression of (c) above. [3]

(e) Implement the circuit of the following function using NAND gates only. Use as few gates as possible. [5]

$$f(a,b,c) = \overline{ab} + \bar{a}\cdot c$$

Question 4

(a) With an aid of a diagram, describe the operation of a D-latch. [5]

(b) What is the difference between a full adder and a half adder? [2]

(c) Draw a circuit that inputs a four bit number. The circuit outputs 1 if the input number is any of the following numbers: 2, 3, 10, 11, 12 and 15. Otherwise, it outputs a 0. [10]

(d) Explain the difference between exclusive OR and inclusive OR gates. [2]

(e) Find the canonical POS of the function $f(a,b,c) = (a+\bar{b})\cdot\bar{c}$ [5]

(f) Give an example of a sequential circuit. [1]

Question 5

(a) Define a predicate and give an example. [2]

(b) Represent the following statements using predicate logic. Assume the universal set as all human beings.

(i) All cats have fur and only a few sleep at night. [2]

(ii) Some parents are married and take care of their children and all divorced parents fight for child support. [3]

(c) Use universal instantiation and inference rules to prove the following; [5]

No cat can catch Jerry.

Tom is a cat.

Therefore, Tom cannot catch Jerry.

(d) Rewrite the following using the universal quantifier only. [3]

$\forall x(\exists y(Loves(x, y)))$

(e) What is the possible meaning of the predicate statement in (d) above? [2]

(f) Prove that $\forall x\big(P(x) \wedge Q(x)\big) \equiv \forall x P(x) \wedge \forall x Q(x)$ [4]

(g) Explain the limitation of propositional logic that is solved by predicate logic. Use an example. [4]