UNIVERSITY OF ESWATINI

Department of Computer Science

NOVEMBER/DECEMBER MAIN EXAMINATION

COURSE TITLE : DISCRETE MATHEMATICS

COURSE CODE : CSC203

TOTAL MARKS :100

DURATION OF EXAM : THREE (3) HOURS

NUMBER OF EXAM PAGES: 5 (includes cover page)

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Answer all questions

QUESTION ONE {40 marks}

1.1) Let f(x) = 3x – 2. What is its inverse? {3 marks}

1.2) Let f(x) = 2x – 1, g(x) = 3x, and h(x) = x^2 + 1. Compute the following:

(i) f(g(-3)) {3 marks}

(ii) (ii) f(h(7)) {3 marks}

(iii) (iii) g(h(24)) {3 marks}

1.3) Let $\{a_n\}$ be a sequence that satisfies the recurrence relation $a_n = a_{n-1} + 3$ for n = 1,2,3,4,.... and suppose that $a_0 = 2$.

[Here a0 = 2 is the initial condition.]

What are a_1 , a_2 and a_3? {3 marks}

1.4) The Fibonacci *sequence is defined by* $f_0, f_1, f_2, \ldots$, where:

Initial Conditions: $f_0 = 0, f_1 = 1$

Recurrence Relation: $f_n = f_{n-1} + f_{n-2}$

Find f_2, f_3, f_4, f_5 and f_6 . {5 marks}

1.5) Given the matrices A and B,

Show that $B = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$ is the inverse of $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$.

{4 marks}

1.6) An arithmetic progression has 3 as its first term. Also, the sum of the first 8 terms is twice the sum of the first 5 terms. Find the common difference. {3 marks}

1.7) Find the sum of the first five terms of the GP with first term 3 and common ratio 2. {3 marks}

1.8) How many different license plates can be made if each plate contains a sequence of three uppercase English letters followed by three digits? {3 marks}

1.9) A group of 30 people have been trained as astronauts to go on the first mission to Mars. How many ways are there to select a crew of six people to go on this mission? {3 marks}

1.10) A farmer purchased 3 cows, 2 pigs, and 4 hens from a man who has 6 cows, 5 pigs, and 8 hens. Find the number m of choices that the farmer has. {4 marks}

QUESTION TWO {20 marks}

2.1) Show that $(p \land q) \to (p \lor q)$ is a tautology.

(a) Using logical equivalences. {4 marks}

(b) Using truth tables. {4 marks}

2.2) Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English.

(2.2.1) The file system cannot be backed up if there is a user currently logged on. {3 marks}

(2.2.2) There are at least two paths connecting every two distinct endpoints on the network. {3 marks}

(2.2.3) No one knows the password of every user on the system except for the system administrator, who knows all passwords. {3 marks}

2.3) As mentioned in the textbook, the notation $\exists!xP(x)$ denotes "There exists a unique x such that P(x) is true." If the domain consists of all integers, what is the truth value of the statement $\exists!x(x + 3 = 2x)$ Justify your answer. {3 marks}

QUESTION THREE {25 marks}

3.1) Use mathematical induction to prove that $n(n + 5)$ is divisible by 2 for any positive integer n. {5 marks}

3.2) Prove by contradiction the following:
For all integers n, if n^2 is odd, then n is odd. {4 marks}

3.3) For any sets A, B and C, prove the following (Do not use truth tables):
$(A \cup B) \cup C = A \cup (B \cup C)$ {6 marks}

3.4) Let A = {1, 2, 3, 4} and R = {(1, 1), (1, 3), (2, 2), (2, 4), (3, 1), (3, 3), (4, 2), (4, 4)}.

Show that R is an Equivalence Relation. {6 marks}

3.5) Determine the value of the following:

3.5.1) 35 (MOD 7) {2 marks}

3.5.2) 20 (MOD 3) {2 marks}

QUESTION FOUR {15 marks}

4.1) Using the Pigeonhole Principle, find the minimum number of student's in a class to be sure that three of them are born in the same month. {2 marks}

4.2) If A and B are two mutually exclusive events, then $P(A \cup B)=P(A)+P(B)$

Two dice are tossed once. Find the probability of getting an even number on first dice or a total of 8. {4 marks}

4.3) Consider the graph shown in Fig 1. Give an example of the following:

4.3.1) An elementary path from V_1 to V_6. {3 marks}

4.3.2) A simple path which is not elementary from V_1 to V_6. {3 marks}

4.3.3) A circuit which is not simple and starting from V_2. {3 marks}

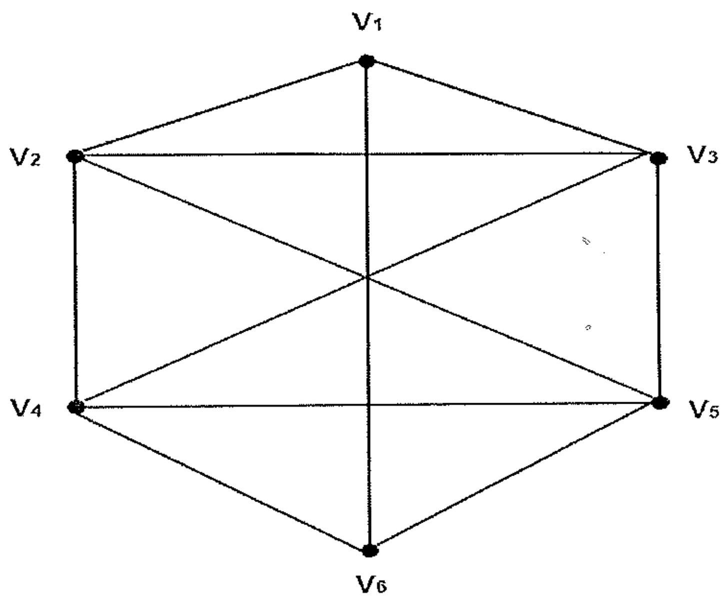

Fig 1