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UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRONIC ENGINEERING

MAIN EXAMINATION: 2005

TITLE OF THE PAPER: LINEAR SYSTEMS

COURSE NUMBER: E352

TIME DURATION: THREE HOURS

INSTRUCTIONS: Chose and attempt FOUR QUESTIONS. Each question carries 25 Marks.

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This paper contains **Eight pages** including this one

Question 1

a) Explain the following terms:

causal system; discrete-time system.

Give an example in each case. (4 Marks)

b) Consider the circuit shown in Fig 1.2.

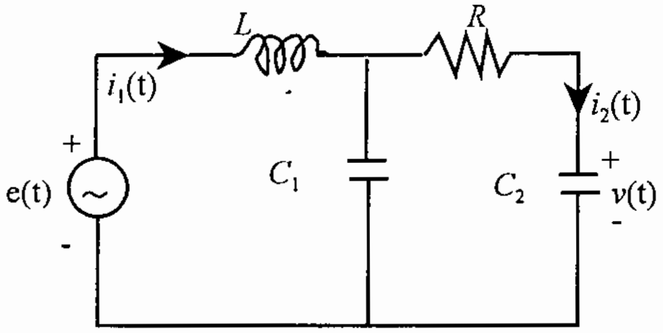


Fig 1.1

Use Kirchhoff's voltage law to determine the system operator T(p) given as

$$i_2(p) = T(p)e(p)$$

(13 Marks)

c) (i) A 0-100 °C thermometer is found to have a constant error of +0.2 °C. Determine the percentage error at readings of 10 °C, 50 °C and 100 °C. Comment on the results (5 marks)

(ii) A vibrating measuring system involves the use of a piezo-electric transducer, a charge amplifier, and a UV recorder. If the maximum errors are ±0.5%, ±1.0% and ±1.5%, respectively, calculate the maximum possible system error and the probable error. What would be the experimental error you would present in your laboratory report for such an experiment?

(3 Marks)

Question 2

a) A discrete-time system is given in Fig 2.1 as a block diagram. Assume $y_k=0$ for $k\leq 0$.

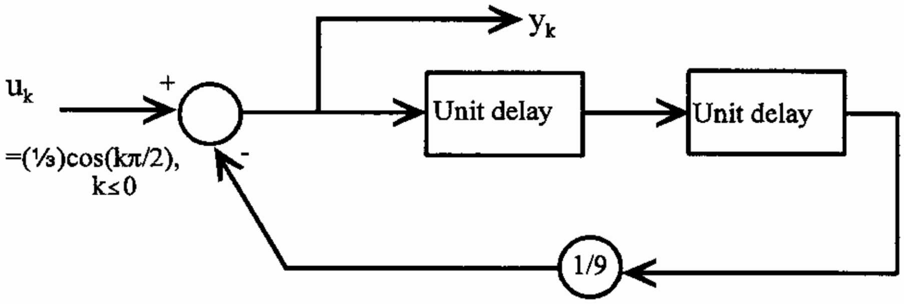


Fig 2.1

(i) Write down the corresponding difference equation. (2 Marks)

(ii) Find the homogenous solution. (3 Marks)

(iii) Using the annihilator technique, show that the general solution is

$$y_k = C_1\left(\frac{1}{3}\right)^k \cos\frac{k\pi}{2} + C_2\left(\frac{1}{3}\right)^k \sin\frac{k\pi}{2} + \frac{1}{2}\left(\frac{1}{3}\right)^k \cos\frac{k\pi}{2}$$ (11 Marks)

(iv) Evaluate the arbitrary constants C_1 and C_2 to find the actual output sequence. Note that $y_{-1} = y_{-2} = 0$. (6 Marks)

b) Consider the second-order difference equation.

$$y_k = 2ay_{k-1} - y_{k-2}$$

Draw an associated block diagram. (3 Marks)

Question 3

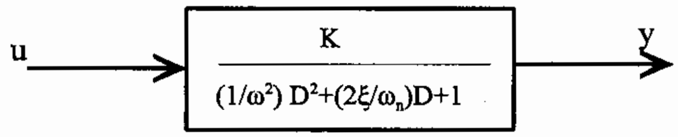
a) After applying Kirchhoff's voltage law to an electric circuit loop the following equation was established

$$\frac{dv(t)}{dt} + 5v(t) = 500e^{-2t}$$

with $v(0)=20V$.

Use the integrating factor technique to find v(t) in volts. (7 Marks)

b) (i) A second-order system may be represented in block form as



where u is the input and y the output. K is a constant and D an operator. Briefly explain ω_n and ξ and their use in system characterization. (9 Marks)

(ii) A mass-spring damper system has a first overshoot of approximately 40% of its final value when subjected to a step input force. Estimate the values of ξ and ω_n if the time taken to reach the first overshoot is 0.8s from the application of the step.[Use Figs 3.1 and 3.2 at the end of the paper.]

(3 Marks)

c) The differential equation describing a mercury-in-glass thermometer is

$$4\frac{dH}{dt} + 2H = 2x10^{-3}T$$

where H is the height of mercury in meters and T is the input temperature in °C. Determine the time constant and the static sensitivity of the thermometer. (6 Marks)

Question 4

a) (i) Explain what a block diagram is. What is its usefulness? (7 Marks)

(ii) Draw two block diagrams in parallel and their equivalent diagram. (2 Marks)

b) (i) Consider the block diagram of armature controlled d.c. motor (Fig 4.1). What is the overall transfer function?

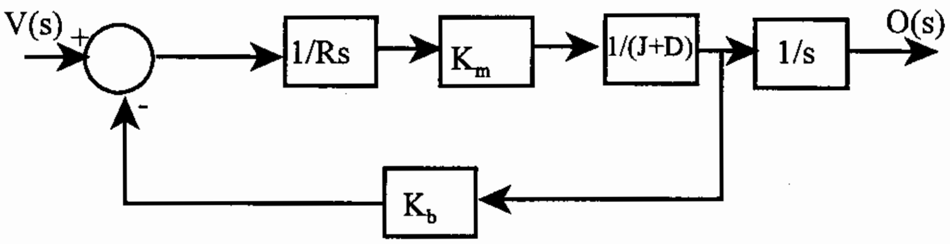


Fig 4.1

(8 Marks)

(ii) Simplify the block diagram shown in Fig 4.2 and hence determine the system transfer function

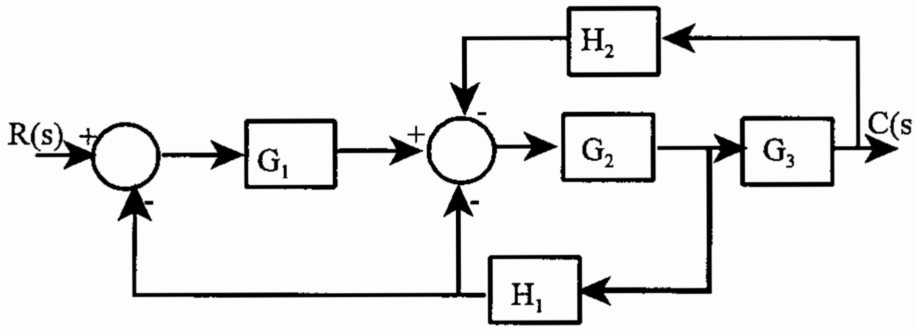


Fig 4.2

(8 Marks)

Question 5

a) A measuring system can be represented by a spring, mass and damper as shown in Fig 5.1.

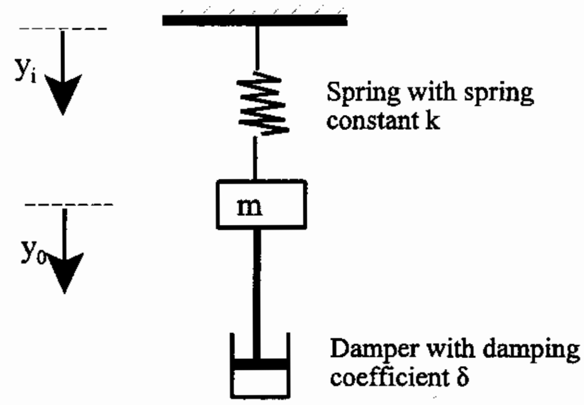


Fig 5.1

Show that the transfer function is given by

$$\frac{y_o(p)}{y_i(p)} = \frac{1}{\frac{m}{k}p^2 + \frac{\delta}{k}p + 1}$$

Express the coefficients of p^2 and p in terms of standard quantities: damping ratio and angular frequency. (10 Marks)

b) A second-order system is described by the differential equation

$$0.1\frac{d^2y}{dt^2} + 0.2\frac{dy}{dt} + 2.5y = 2.8x$$

where x is the input and y is the output.

(i) Determine the system undamped natural frequency in Hz. (5 Marks)

(ii) What is the damping ratio of the system? (4 Marks)

c) Consider a seismic mass accelerator with a natural frequency of 250 Hz and a damping ratio of 0.7. Determine the damped natural frequency and the time for the output to decay to 1% of the original value. (6 Marks)

Question 6

a) (i) Find the inverse Laplace transform (do not use tables) of

$$\frac{e^{-3s}}{s^3}$$

Give a graphical representation. (5 Marks)

(ii) Find the Laplace transform of the function shown in Fig 6.1.

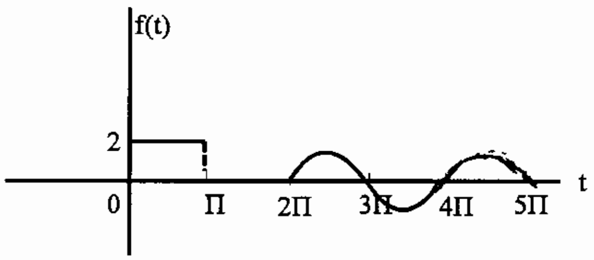


Fig 6.1

(6 Marks)

b) A linear system is described by the following differential equation. This system is forced with an input shown in Fig 6.2. Find the output of the system.

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = u(t)$$

$$y(0) = 0, \quad \frac{d}{dt}y(0) = 1$$

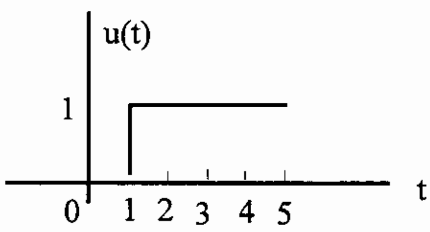


Fig 6.2

(14 Marks)

Standard Response Curves

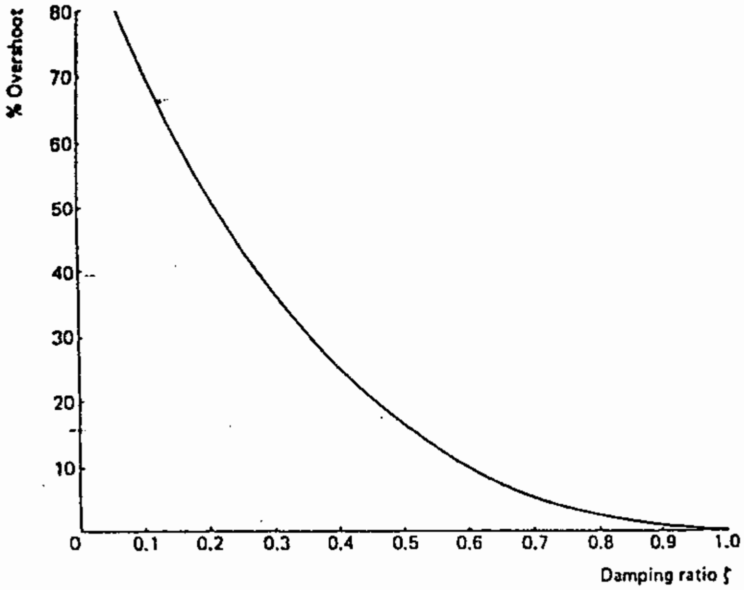


Fig 3.1 Graph of % overshoot against damping ratio ζ

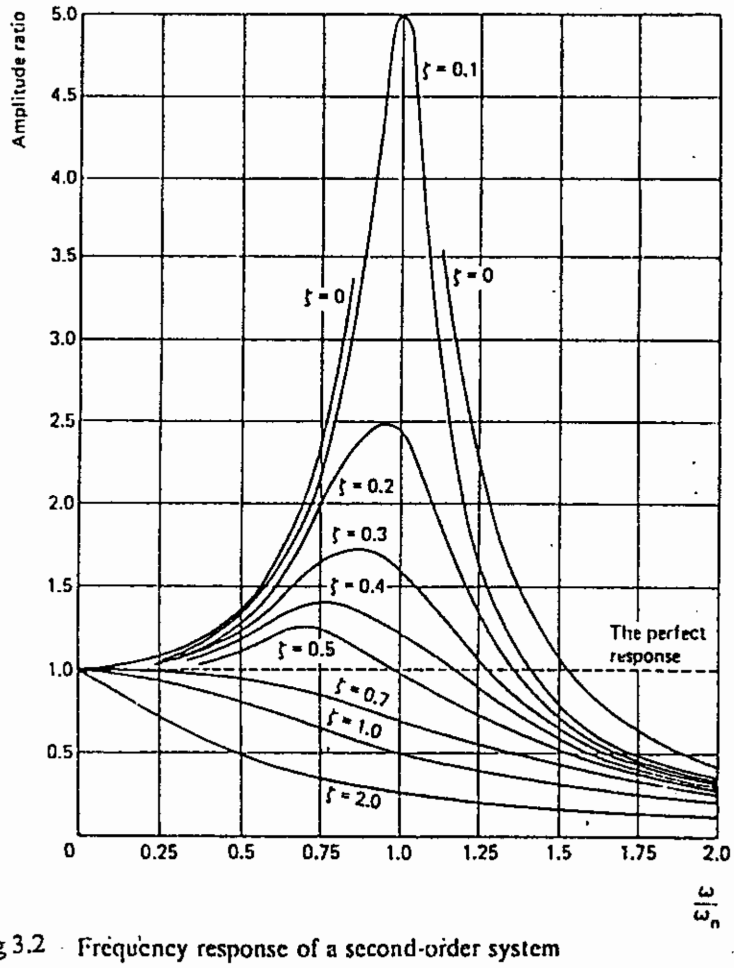


Fig 3.2 Frequency response of a second-order system

Laplace Transform Table

$f(t)$	$F(s)$	Convergence Region
1. $e^{-at}\xi(t)$	$\frac{1}{s+a}$	$-\text{Re}(a) < \text{Re}(s)$
2. $\xi(t)$	$\frac{1}{s}$	$0 < \text{Re}(s)$
3. $t\,\xi(t)$	$\frac{1}{s^2}$	$0 < \text{Re}(s)$
4. $t^n\,\xi(t)$	$n!/s^{n+1}$	$0 < \text{Re}(s)$
5. $\delta(t)$	1	All s
6. $\delta^{(1)}(t)$	s	All s
7. $\text{sgn}\, t$	$\frac{2}{s}$	$\text{Re}(s) = 0$
8. $-\xi(-t)$	$\frac{1}{s}$	$\text{Re}(s) < 0$
9. $te^{-at}\xi(t)$	$\frac{1}{(s+a)^2}$	$-\text{Re}(a) < \text{Re}(s)$
10. $t^n e^{-at}\xi(t)$	$\frac{n!}{(s+a)^{n+1}}$	$-\text{Re}(a) < \text{Re}(s)$
11. $e^{-a\|t\|}\xi(t)$	$\frac{2a}{a^2-s^2}$	$-\text{Re}(a) < \text{Re}(s) < \text{Re}(a)$
12. $(1-e^{-at})\,\xi(t)$	$\frac{a}{s(s+a)}$	$\max[0, -\text{Re}(a)] < \text{Re}(s)$
13. $\cos\omega t\,\xi(t)$	$\frac{s}{s^2+\omega^2}$	$0 < \text{Re}(s)$
14. $\sin\omega t\,\xi(t)$	$\frac{\omega}{s^2+\omega^2}$	$0 < \text{Re}(s)$
15. $e^{-\sigma t}\cos\omega t\,\xi(t)$	$\frac{s+\sigma}{(s+\sigma)^2+\omega^2}$	$-\sigma < \text{Re}(s)$
16. $e^{-\sigma t}\sin\omega t\,\xi(t)$	$\frac{\omega}{(s+\sigma)^2+\omega^2}$	$-\sigma < \text{Re}(s)$
17. $\begin{cases} 1-\|t\|, & \|t\| < 1 \\ 0, & \|t\| > 1 \end{cases}$	$\left(\frac{\sinh s/2}{s/2}\right)^2$	All s
18. $\sum_{n=0}^{\infty}\delta(t-nT)$	$\frac{1}{1-e^{-sT}}$	All s