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UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRONIC ENGINEERING

MAIN EXAMINATION 2005

TITLE OF PAPER	:	MATHEMATICAL METHODS I (PAPER ONE)
COURSE NUMBER	:	E370(I)
TIME ALLOWED	:	THREE HOURS
INSTRUCTIONS	:	ANSWER ANY FOUR OUT OF FIVE QUESTIONS.
		EACH QUESTION CARRIES 25 MARKS.
		MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.

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E370(I) MATHEMATICAL METHODS I (PAPER ONE)

Question one

The following non-homogeneous differential equation represents a simple harmonic oscillator of mass $m = 2\ kg$ and spring force constant $K = 17\ \frac{N}{m}$ forced to oscillate in an viscous fluid :

$$2\frac{d^2 x(t)}{d t^2} + 6\frac{d x(t)}{d t} + 17\, x(t) = f(t)$$

where $x(t)$: displacement from its resting position

$6\frac{d x(t)}{d t}$: retardation force by the viscous fluid

$f(t)$: externally applied driving force

(a) Find and write down the general solution to the homogeneous part of the above given differential equation , i.e., $2\frac{d^2 x(t)}{d t^2} + 6\frac{d x(t)}{d t} + 17\, x(t) = 0$ (5 marks)

(b) If the driving force is given as $f(t) = 9\sin(2t)$, set the particular solution of the given non-homogeneous differential equation as $x(t) = k_1 \cos(2t) + k_2 \sin(2t)$ and find the values of k_1 *and* k_2 , (10 marks)

(c) (i) Combine the obtained solutions in (a) and (b) to write down the general solution of the given non-homogeneous differential equation , (2 marks)

Question one (continued)

(ii) If the given initial conditions for the system are $x(0) = 4$ and $\left.\dfrac{d\,x(t)}{d\,t}\right|_{t=0} = 0$, find the values of the arbitrary constants in (c)(i) and thus the specific solution for the given system. (5 marks)

(iii) Plot the specific solution of $x(t)$ obtained in (c)(ii) for $t = 0$ *to* 10 sec . (3 marks)

Question two

Given the following differential equation $\dfrac{d^2 y(x)}{d x^2} + 16\, y(x) = 0$

(a) By direct substitution, show that $\sin(4x)$ *and* $\cos(4x)$ are solutions to the given differential equation. (3 marks)

(b) Set $y(x) = \sum_{n=0}^{\infty} a_n x^{n+s}$ *and* $a_0 \neq 0$, use the power series method to

(i) find the indicial equations and thus deduce that $s = 0$ *or* 1 and $a_1 = 0$,

(7 marks)

(ii) find the recurrence relation, (4 marks)

(iii) for $s = 1$ and $a_1 = 0$, set $a_0 = 1$ and find the values of a_2 , a_3, ... , a_{10} by using the recurrence relation in (b)(ii), (6 marks)

(iv) plot both $y(x) = (\sum_{n=0}^{10} a_n x^{n+1})$ with those a_n values obtained in (b)(iii) and $\sin(4x)$ for $x = 0$ *to* 1 and show them in a single display . Make a brief remark from your diagram. (5 marks)

Question three

(a) Given $m \dfrac{d^2 x}{d t^2} = - k x$, and $m = 2 \ kg$ & $k = 50 \ \dfrac{N}{m}$

(i) find the values of the angular frequency , frequency and period of the given simple harmonic oscillator system , (3 marks)

(ii) write down the general solution of the given problem . (2 marks)

(b) Two simple harmonic oscillators (one is represented by m_1 *and* k_1 and the other represented by m_2 *and* k_2) are jointed together by a spring of spring constant K . The equations of motion for the system are :

$$\begin{cases} m_1 \dfrac{d^2 x_1(t)}{dt^2} = -(k_1 + K) x_1(t) + K x_2(t) \\ m_2 \dfrac{d^2 x_2(t)}{dt^2} = K x_1(t) - (k_2 + K) x_2(t) \end{cases}$$

where $x_1(t)$ and $x_2(t)$ are the displacement from their respective resting position .

If $m_1 = 1 \ kg$, $m_2 = 2 \ kg$, $k_1 = 2 \ \dfrac{N}{m}$, $k_2 = 4 \ \dfrac{N}{m}$ and $K = 16 \ \dfrac{N}{m}$,

(i) show that the coupled differential equations for the system can be simplified to be :

$$\begin{cases} \dfrac{d^2 x_1(t)}{d t^2} = -18 x_1(t) + 16 x_2(t) \\ \dfrac{d^2 x_2(t)}{d t^2} = 8 x_1(t) - 10 x_2(t) \end{cases}$$

(2 marks)

Question three (continued)

(ii) set $x_1(t) = X_1 e^{i\omega t}$ and $x_2(t) = X_2 e^{i\omega t}$, showing deduction details , find the eigenfrequencies ω of the given coupled system , (6 marks)

(iii) find the eigenvectors of the given coupled system corresponding to each eigenfrequencies found in (b)(ii), (6 marks)

(iv) find the normal coordinates of the given coupled system corresponding to each eigenfrequencies found in (b)(ii) . (6 marks)

Question four

(a) Given the differential equation for a damped oscillator system as

$$\frac{d^2\,y(t)}{d\,t^2} + 6\,\frac{d\,y(t)}{d\,t} + 25\,y(t) = 0$$ and its initial conditions as $y(0) = 10$ and $y'(0) = 5$

(i) Use *dsolve* command to find its specific solution for the system and plot it for $t = 0\ to\ 3\ \sec$ (5 marks)

(ii) Find the Laplace transform of $y(t)$, i.e., $Y(s)$, (5 marks)

(ii) find the inverse Laplace transform of $Y(s)$, i.e., $y(t)$, and compare it with the solution obtained in (a)(i). (3 marks)

(b) Given the Laplace transform of $f(t)$ and $g(t)$ as $F(s) = \dfrac{1}{s^2 - 2s - 8}$ and $G(s) = \dfrac{s+1}{s^2}$ respectively,

(i) find $f(t)$ and $g(t)$, (3 marks)

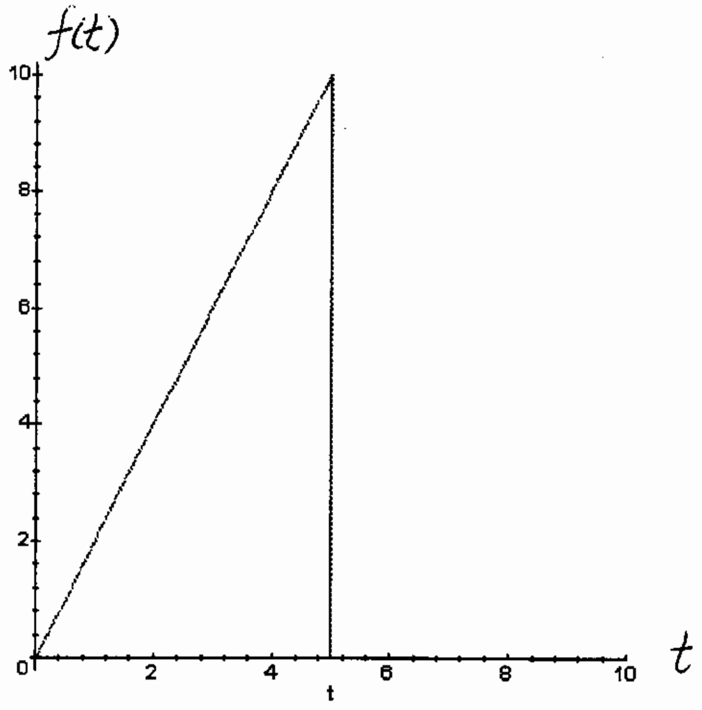
(ii) using the t - shift theorem, find the inverse Laplace transform of $e^{-2s}\,F(s)$. Plot it for $t = 0\ to\ 3\ \sec$, (4 marks)

(iii) using the convolution theorem, find the inverse Laplace transform of $F(s)\,G(s)$. Also use *invlaplace* command directly to find the answer. Compare both answers and make a brief comment. (5 marks)

Question five

Given the differential equation for a forced oscillator system as

$$\frac{d^2 y(t)}{dt^2} + 4\, y(t) = f(t)$$, where the external force $f(t)$ is given as



and the initial conditions are given as $y(0) = 0$ and $y'(0) = 0$

(a) (i) express $f(t)$ in terms of Heaviside functions and then plot it for

$t = 0$ *to* 10 to reproduce the given graph, (5 marks)

(ii) find the Laplace transform of $f(t)$, i.e., $F(s)$, by integration,

(4 marks)

Question five (continued)

(b) (i) Find the laplace transform of $y(t)$, i.e., $Y(s)$, and show that

$$Y(s) = G(s)\,F(s) \quad \text{where} \quad G(s) = \frac{1}{s^2 + 4} \,,$$ (4 marks)

(ii) find the inverse laplace transform of $Y(s)$, i.e., $y(t)$, and plot it for $t = 0 \;\; to \;\; 10$, (5 marks)

(iii) find the inverse laplace transform of $G(s)$, i.e., $g(t)$, and then find $y(t)$ by convolution of $g(t)$ and $f(t)$. Express $y(t)$ explicitly for $0 \le t \le 5$ and $5 \le t$ intervals. Plot this piecewise $y(t)$ for $t = 0 \;\; to \;\; 10$ and compare it with that plotted in (b)(ii). (7 marks)