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UNIVERSITY OF SWAZILAND

MAIN EXAMINATION 2004/2005

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRONIC ENGINEERING

TITLE OF PAPER: DIGITAL SIGNAL PROCESSING

COURSE CODE: E420

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

1. **Answer any <u>FOUR</u> (4) of the following six questions.**
2. **Each question carries 25 marks.**
3. **Tables of selected window functions and z-transforms are attached.**

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THIS PAPER CONTAINS NINE (9) PAGES INCLUDING THIS PAGE

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Q.1 (a) Your local supermarket has hired you to design a digital weighing scale to weigh vegetables and fruit items as they pass over a bar code scanner. The bar code consists of a description of the type of item only (e.g. oranges from X, potatoes from Y etc.) without its weight or price. The digital scale weighs and prices the item and communicates this to the cash register.

(i) Draw a labeled block diagram of an appropriate system based on DSP principles that accomplishes the above task. *(10 marks)*

(ii) Is the system linear time-invariant? Explain your answer. *(3 marks)*

(b) A discrete linear time-invariant system has a response *y(n)* to a unit step response *u(n)*, given by

$$y(n) = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \text{ and } n \text{ even (i.e. } n = 0,2,4,6, \ldots) \\ 0, & n > 0, \text{ and } n \text{ odd (i.e. } n = 1,3,5,7, \ldots) \end{cases}$$

Assume that the system is initially at rest.

(i) Determine the unit impulse response h(n) of the system. You may use the fact that $\delta(n) = u(n) - u(n-1)$. *(6 marks)*

(ii) Determine and sketch the first six terms of the output when the input is

$$x(n) = \begin{cases} 1, & n = 0 \\ -1, & n = 1 \\ 0, & n \text{ otherwise} \end{cases}$$

(6 marks)

Q.2 (a) Evaluate and plot the frequency response of the discrete system in Fig. Q.2a. Assume that the sampling frequency is 10 Hz and that all branch gains are 1 unless otherwise stated. *(13 marks)*

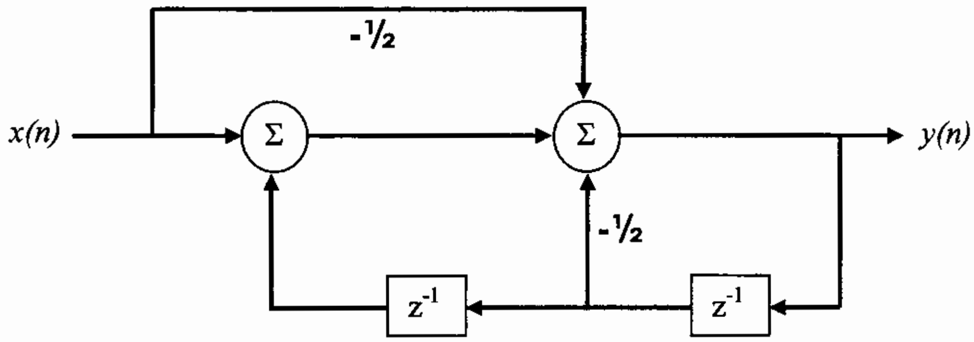

Fig. 2a

(b) Figure Q.2b shows the z-plane representation of a filter with one zero and one pole.

(i) Sketch its amplitude response in the range 0 to 0.5 of the sampling frequency. *(5 marks)*

(ii) Derive an equation for the magnitude response of the filter. *(6marks)*

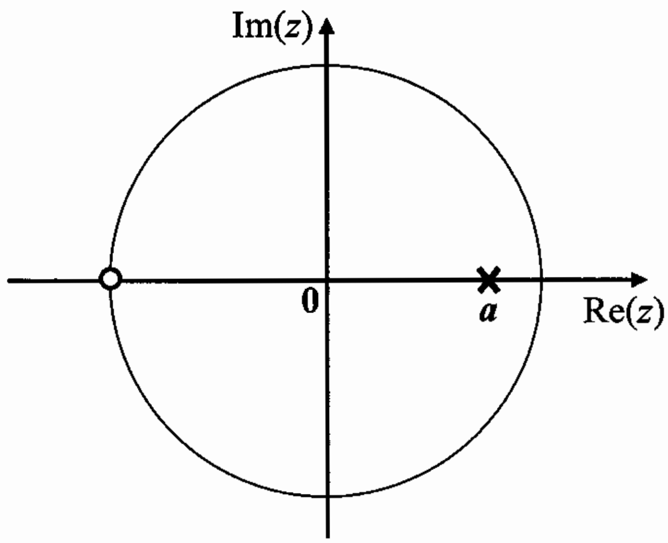

Fig. Q2b

Q.3 (a) Explain the principles behind the Fast Fourier Transform Algorithm.

(8 marks)

(b) For an eight-point signal in time demonstrate how a DFT may be converted into an FFT and show how this reduces the number of complex multiplications required. *(7 marks)*

(c) For a four-point DFT, it is suggested that you use a window function $w(n) = [0.5, 1, 1, 0.5]$ on the data. Demonstrate whether this is better than using a simple rectangular function $w(n) = [1, 1, 1, 1]$. *(10 marks)*

Q.4 (a) An analogue prototype transfer function $H(s) = \dfrac{s}{s-0.5}$ is used to find an equivalent low pass digital filter whose gain is -3 dB at 400 Hz and sampling frequency is 4 kHz. Using the Bilinear Z-Transform:

(i) Derive the transfer function of the digital filter. *(13 marks)*

(ii) Find the difference equation relating its output to the input sequence. *(2 marks)*

(b) A linear-phase filter has a double zero at $z = -1$ and a conjugate pair of zeros at $0.5(1 \pm j\sqrt{3})$. Derive a stable, causal real impulse response *h(n)* for this filter. Assume that the d.c. gain is 1. *(10 marks)*

Q.5 (a) (i) Explain the principles behind the auto- and cross-correlation. Briefly discuss two applications which use these functions.

(7 marks)

(ii) Three signals are given as follows:

$$s_1(n) = [0, 0, 0, 0, 1, 1, 0, 0, 0, 0]$$

$$s_2(n) = [0, 1, 1, 0]$$

$$s_3(n) = [0, 1, -1, 0]$$

Calculate, compare and comment on the cross-correlations of:

$s_1(n)$ and $s_2(n)$

$s_1(n)$ and $s_3(n)$. *(4 marks)*

(b) What different number-representations are available within DSP chips? Give an example of each, commenting on their advantages and disadvantages.

(7 marks)

(c) Describe the main characteristics of the von Newmann and the Harvard architectures as used in microprocessor or DSP chips. What advantages does one have over the other? *(7 marks)*

Q.6 (a) Comment on the following statement:

"The resolution of a DFT may be increased by taking more samples and/or increasing the sampling period."

(6 marks)

(b) Using the window method with a Hanning window, calculate the coefficients of a 9-tap linear-phase FIR lowpass digital filter. The filter is to have a cutoff frequency of 4 kHz and sampling frequency is 20 kHz. Your calculations are expected to be accurate to 4 decimal places.

(19 marks)

== END OF EXAMINATION PAPER. ATTACHMENTS FOLLOW ==

TABLE OF Z-TRANSFORMS OF SOME COMMON SEQUENCES

Discrete-time sequence $x(n),\ n \geq 0$	**Z-transform** $H(z)$
$k\delta(n)$	k
k	$\frac{kz}{z-1}$
$ke^{-\alpha n}$	$\frac{kz}{z-e^{-\alpha}}$
$k\alpha^n$	$\frac{kz}{z-\alpha}$
kn	$\frac{kz}{(z-1)^2}$
kn^2	$\frac{kz(z+1)}{(z-1)^3}$
$kn\alpha^n$	$\frac{k\alpha z}{(z-\alpha)^2}$

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SUMMARY OF IMPORTANT FEATURES OF SELECTED WINDOW FUNCTIONS

Name of Widow	Normalized Transition Width	Passband Ripple (dB)	Main lobe relative to Sidelobe (dB)	Max. Stopband attenuation (dB)	6 dB normalized bandwidth (bins)	Window Function $\omega(n)$, $\lvert n\rvert \le (N-1)/2$
Rectangular	0.9/N	0.7416	13	21	1.21	1
Hanning	3.1/N	0.0546	31	44	2.00	$0.5+0.5\cos\left(\frac{2\pi n}{N}\right)$
Hamming	3.3/N	0.0194	41	53	1.81	$0.54+0.46\cos\left(\frac{2\pi n}{N}\right)$
Blackman	5.5/N	0.0017	57	74	2.35	$0.42+0.5\cos\left(\frac{2\pi n}{N-1}\right)+0.08\cos\left(\frac{4\pi n}{N-1}\right)$
Kaiser	2.93/N (β=4.54)	0.0274		50		$\frac{I_o\left(\beta\left\{1-\left[\frac{2n}{N-1}\right]^2\right\}^{\frac{1}{2}}\right)}{I_o(\beta)}$
	4.32/N (β=6.76	0.00275		70		
	5.71/N (β=8.96)	0.000275		90		

$$\text{Bin width} = \frac{f_s}{N}\ \text{Hz}$$