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UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION 2004/2005

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRONIC ENGINEERING

TITLE OF PAPER: SIGNALS II

COURSE NUMBER: E462

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

1. **Answer any <u>FOUR</u> (4) of the following FIVE questions.**
2. **Each question carries 25 marks.**
3. **Tables of selected Fourier transform pairs and Fourier transform properties are attached.**

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THIS PAPER CONTAINS EIGHT (8) PAGES INCLUDING THIS PAGE

Question One (25 marks)

(a) Consider a periodic pulse wave form with on time T_p and period T_o, ($T_o > T_p$). The pulse is centred around $t = 0$ sec.

(i) Obtain its complex Fourier series expansion. *(8 marks)*

(ii) Sketch the amplitude of the Fourier Coefficients over the range

$$-\frac{1}{T_p} \leq f \leq \frac{1}{T_p}$$ *(5 marks)*

(iii) If $T_p = 4\ \mu s$ and $T_o = 50$ ms, calculate the number of spectral lines existing in the range given in (ii) above. *(4 marks)*

(b) Show that convolution of two signals in the time domain is equivalent to multiplication in the frequency domain. *(8 marks)*

Question Two (25 marks)

(a) If $e^{-t+3}u(t-3) * h(t) = 2e^{-t}u(t)$, find *h(t)* using a Fourier Transform method.

(8 marks)

(b) Use the Fourier transform tables/properties to find the inverse Fourier transforms of the following:

(i) $H(j\omega) = j\delta(\omega + 50\pi) - j\delta(\omega - 50\pi)$ *(5 marks)*

(ii) $H(j\omega) = \dfrac{1}{6 + \omega^2 + j\omega}$ (Hint: First use partial fraction expansion)

(7 marks)

(iii) $H(j\omega) = 2(j\omega)^2 \pi\delta(\omega + \omega_o)$ (Hint: Use differentiation property) *(5 marks)*

Question Three (25 marks)

(a) (i) What is the relationship between the instantaneous frequency and phase of a sinusoidal signal? *(2 marks)*

(ii) A chirp signal of amplitude 5 is generated by sweeping linearly from 2500 Hz to 500 Hz. The initial phase of the signal is 60 deg and 2 sec after the beginning of the sweep its frequency is 1000 Hz. Obtain a mathematical expression for this signal. *(11 marks)*

(b) Obtain the spectrum of the time-limited sinusoidal signal sketched in Fig. Q.3. Hint: You should first obtain a sinusoidal expression to represent the signal assuming that it was not time limited and then use the limits in your integral. *(12 marks)*

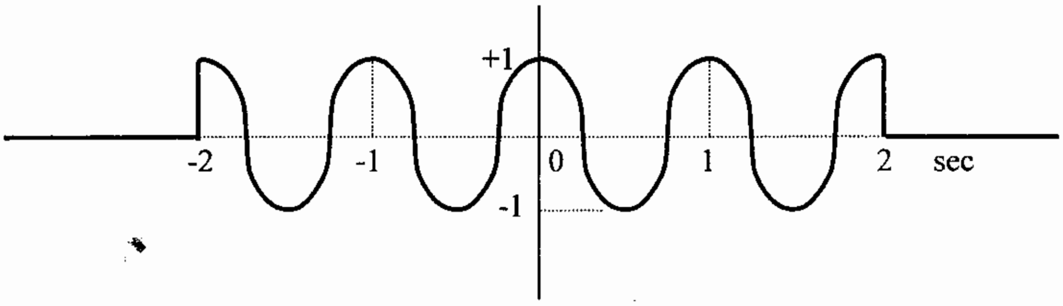


Fig. Q.3

Question Four (25 marks)

(a) Using the definition of power of a signal, find the power in the signal

$$x(t) = 2\cos t + 5\cos 2t, \quad \text{where } -\infty < t < \infty$$ *(6 marks)*

(b) Using the definition of the energy of a signal, find the energy in the signal

$$x(t) = e^{-|t|}, \quad \text{where } -\infty < t < \infty$$ *(6 marks)*

(c) A signal is described by

$$x(t) = 6\sin 20\pi t + 4\cos 40\pi t + 3\cos 60\pi t$$

(i) Sketch its magnitude and phase spectra. *(6 marks)*

(ii) A single-pole low pass filter with a cut off frequency equal to twice the fundamental frequency is used to filter the signal. Find the total power of the filtered signal. *(7 marks)*

Question Five (25 marks)

(a) Use the properties of a probability density function (pdf) investigate whether the following functions are valid pdf 's or not:

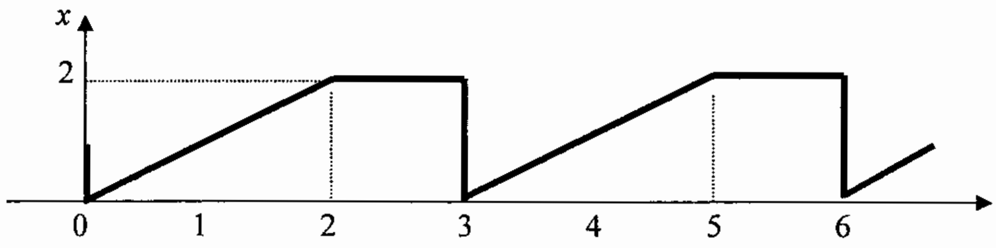
(i) $p(x) = 0.3\delta(x+1) + 0.2\delta(x) + 0.5\delta(x-1)$ *(3 marks)*

(ii) $p(x) = \frac{1}{\pi}\left(\frac{1}{1+x^2}\right)$

Note that $\int \frac{1}{1+x^2} dx = \tan^{-1} x$ *(6 marks)*

(iii) $p(x) = \begin{cases} \frac{1}{2} - \frac{1}{4}|x| , & |x| \leq 2 \\ 0 , & x \text{ otherwise} \end{cases}$ *(6 marks)*

(b) Find and plot the amplitude probability function of the following periodic signal.



(6 marks)

(c) A random variable X has the amplitude probability function,

$$p(x) = \frac{1}{2}e^{-|x|}$$

Find the probability that the amplitude of X is between -2 and +2.

(4 marks)

═ **END OF EXAMINATION PAPER. ATTACHMENTS FOLLOW** ═

TABLE OF BASIC FOURIER TRANSFORM PAIRS

#	Time Domain: x(t)	Frequency Domain: X(jω)
1	$\delta(t)$	1
2	1	$2\pi\delta(\omega)$
3	$\delta(t-t_d)$	$e^{-j\omega t_d}$
4	$e^{j\omega_o t}$	$2\pi\delta(\omega-\omega_o)$
5	$e^{-at}u(t),\ (a>0)$	$\frac{1}{a+j\omega}$
6	$e^{bt}u(-t),\ (b>0)$	$\frac{1}{b-j\omega}$
7	$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$
8	$u(t+\frac{1}{2}T)-u(t-\frac{1}{2}T)$	$\frac{\sin(\omega T/2)}{\omega/2}$
9	$\frac{\sin(\omega_b t)}{\pi t}$	$u(\omega+\omega_b)-u(\omega-\omega_b)$
10	$A\cos(\omega_o t+\phi)$	$\pi Ae^{j\phi}\delta(\omega-\omega_o)+\pi Ae^{-j\phi}\delta(\omega+\omega_o)$
11	$\cos(\omega_o t)$	$\pi\delta(\omega-\omega_o)+\pi\delta(\omega+\omega_o)$
12	$\sin(\omega_o t)$	$-j\pi\delta(\omega-\omega_o)+j\pi\delta(\omega+\omega_o)$
13	$\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_o t}$	$\sum_{k=-\infty}^{\infty} 2\pi a_k\delta(\omega-k\omega_o)$
14	$\sum_{k=-\infty}^{\infty} \delta(t-nT)$	$\frac{2\pi}{T}\sum_{k=-\infty}^{\infty} \delta(\omega-\frac{2\pi}{T}k)$

TABLE OF BASIC FOURIER TRANSFORM PROPERTIES

#	PROPERTY NAME	TIME DOMAIN: x(t)	FREQUENCY DOMAIN: X(jω)
1	Linearity	$ax_1(t)+bx_2(t)$	$aX_1(j\omega)+bX_2(j\omega)$
2	Conjugation	$x^*(t)$	$X^*(-j\omega)$
3	Time-Reversal	$x(-t)$	$X(-j\omega)$
4	Time Scaling	$x(at)$	$\frac{1}{\lvert a \rvert}X(j\frac{\omega}{a})$
5	Time Delay	$x(t-t_d)$	$e^{-j\omega t_d}X(j\omega)$
6	Modulation	$x(t)e^{j\omega_o t}$	$X[j(\omega-\omega_o)]$
7	Modulation	$x(t)\cos(\omega_o t)$	$\frac{1}{2}X[j(\omega-\omega_o)] + \frac{1}{2}X[j(\omega+\omega_o)]$
8	Differentiation	$\frac{d^n x(t)}{dt^n}$	$(j\omega)^n X(j\omega)$
9	Convolution	$x(t)*h(t)$	$X(j\omega)H(j\omega)$
10	Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}X(j\omega)*Y(j\omega)$