UNIVERSITY OF SWAZILAND
FACULTY OF SCIENCE
DEPARTMENT OF ELECTRONIC ENGINEERING

MAIN EXAMINATION 2006

TITLE OF PAPER   :   SIGNALS I

COURSE NUMBER :   E342

TIME ALLOWED   :   THREE (3) HOURS

INSTRUCTIONS   :   ANSWER <u>ANY FOUR</u> OUT OF THE FIVE QUESTIONS

EACH QUESTION CARRIES <u>25</u> MARKS

MARKS FOR DIFFERENT SECTIONS ARE SHOWN
IN THE RIGHT-HAND MARGIN

THIS PAPER HAS 6 PAGES, INCLUDING THIS PAGE

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Question 1

(a) Distinguish between an even signal and an odd signal, illustrating your answer with sketches.   (5 marks)

(b) Determine and plot the discrete-time versions of the signals $x(t) = \cos(2\pi t)$ and $g(t) = \cos(4\pi t)$, both sampled at a rate of 3 Hz. The first sample is taken at time 0. Give only the first 4 samples. What conclusion can be drawn from these signals?   (6 marks)

(c) Define the following terms
    (i) Periodic signal   (2 marks)
    (ii) discrete-time signal   (2 marks)
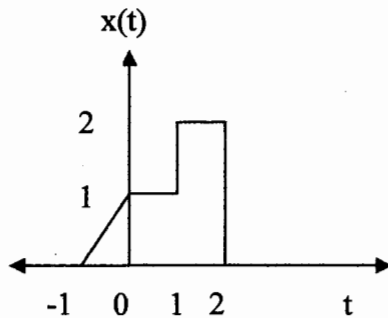
(d) Consider the following signal

$$x(t) = -3\text{rect}(4t - 3)$$

    (i) express it as a sum of even and odd components   (2 marks)
    (ii) sketch the even and odd components   (4 marks)

(e) Given the following continuous time signal $x(t)$

sketch and label each of the following signals
(i) $x(t)u(1-t)$ (2 marks)
(ii) $x(t)\partial(t - 3/2)$ (2 marks)

2

Question 2

(a) Show that the following are aliased signals;

$a(t) = \cos\omega_0 t$ and $b(t) = \cos(\omega_0 + \omega_s)t$,

where $\omega_s = 2\pi f_s$ is the sampling frequency in rad/s   (5 marks)

[*Hint*: $\cos(\alpha + 2n\pi) = \cos(\alpha)$, where n is an integer]

(b) Determine if the following signals are periodic. If periodic specify the fundamental period.
(i) $x(t) = \cos(t) + \sin\sqrt{2}\,t$ (3 marks)
(ii) $x(t) = 2\cos(10t + 1) - \sin(4t - 1)$ (3 marks)
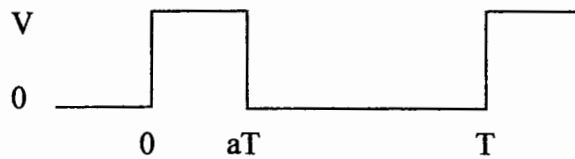
(c) Work out and sketch a well labelled frequency domain representation of the signal

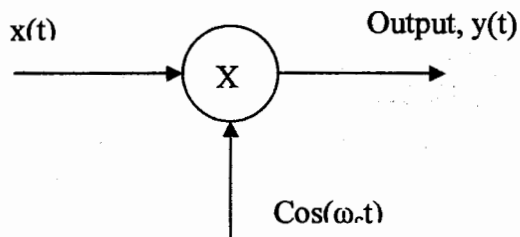$v(t) = 7\cos(5t + 0.8)$ (5 marks)

(d) Determine the root mean square (RMS) value of the following periodic signal
                                                                 (4 marks)

(e) Consider the following system that performs modulation of an carrier signals $\cos(\omega_c t)$ with an input signal $x(t)$ to produce an output, $y(t)$.

Show if the system is
(i) Linear or non-linear   (3 marks)
(ii) Time-invariant or not time-invariant   (2 marks)

3

Question 3

(a) (i) A CD system has a sample rate of 44 KHz. What is the highest frequency that can be sampled without aliasing? (2 marks)

(ii) Samples are to be taken from a record of a continuous-time signal duration 100 ms. The signal contains sinusoidal components with frequencies up to 250 Hz. Determine the minimum number of samples that would be sufficient to give a representation of the signal.

(4 marks)

(b) Plot the magnitude spectrum and phase spectrum of the following periodic signal
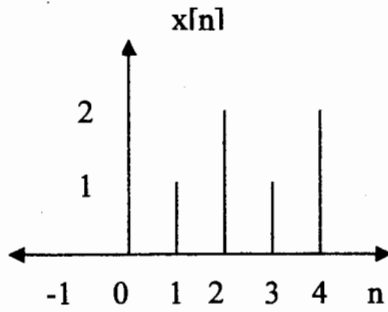
$$x(t) = 1 + \sin(\omega_0 t) + 2\cos(\omega_0 t) + \cos(2\omega_0 t + \pi/4)$$ (12 marks)

Comment on the symmetry of the spectra

(c) Determine if the following signal is periodic or not (4 marks)

$$x(t) = \cos(t) \quad \text{if } t < 0$$
$$= \sin(t) \quad \text{if } t \geq 0$$

(d) With reference to the following signal below, x[n], sketch x[2 – n] (3 marks)

4

Question 4

(a) Verify Parseval's theorem, that is, show that for a periodic waveform

$$P = 1/T_o \int_0^{T_o} |x(t)|^2 \, dt = \sum_{k=-\infty}^{\infty} |C_k|^2$$

and also state the spectral interpretation of the theorem.   (7 marks)

(b) Determine the bandwidth of a signal with the following frequency components:10Hz, 5 Hz, 15 Hz and 50 Hz. What is the required minimum sampling frequency?
(4 marks)

(c) Find the DC term and the first two Fourier coefficients of a unipolar rectangular pulse train with amplitude 3V, period 10 ms, duty cycle 20% and pulse edge at time t = 0.
(14 marks)

Question 5

(a) Sketch and label the following signals and show (by integration) that they are orthogonal

rect(t/2)   and t * rect(t/2)   (5 marks)
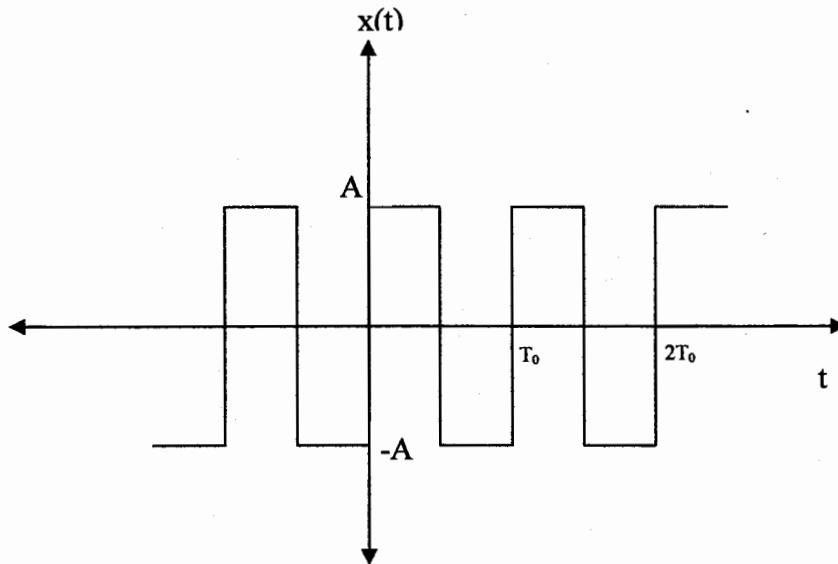
(b) Calculate the average power of the following signal

$x(t) = 7 - 10\cos(40\pi t - \pi/3) + 4\sin(120\pi t)$ (8 marks)

(c) Consider the periodic square wave signal x(t) given below.

Determine its *complex exponential* Fourier series representation   (12 marks)