UNIVERSITY OF SWAZILAND
FACULTY OF SCIENCE
DEPARTMENT OF ELECTRONIC ENGINEERING

SUPPLEMENTARY EXAMINATION 2006

TITLE OF PAPER   :   SIGNALS I

COURSE NUMBER :   E342

TIME ALLOWED :   THREE (3) HOURS

INSTRUCTIONS   :   ANSWER <u>ANY FOUR</u> OUT OF THE FIVE QUESTIONS

                    EACH QUESTION CARRIES <u>25</u> MARKS

                    MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN

THIS PAPER HAS 6 PAGES, INCLUDING THIS PAGE

DO NOT OPEN THE PAPER UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR

Question 1

(a) (i) State the sampling theorem   (2 marks)
    (ii) Distinguish between a power signal and an energy signal   (4 marks)

(b) A pulse signal has the value 1 for $-\tau < t < \tau$ and is zero at all other times. Show that this signal will be orthogonal to all cosine signals with frequencies $\omega = k\pi/\tau$, where k is an integer.
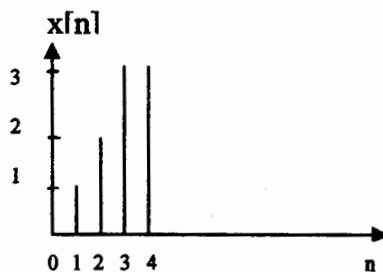
(5 marks)

(c) Sketch the frequency domain representation of the signal

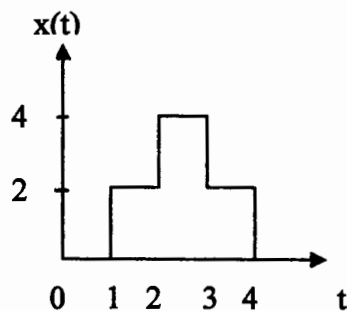 $v(t) = 5\cos(3t + 0.5)$ (5 marks)

(d) Consider the signal given in the figure below

sketch and label each of the following signals

(i) x[2n]   (2 marks)
(ii) x[-n +2]   (2 marks)
(iii) x[n - 2]   (2 marks)

(e)   Sketch the even and odd components of the following signal   (3 marks)

Question 2

(a) Determine if the following signals are periodic. If periodic specify the fundamental period

(i) $x(t) = \cos(t + \pi/4)$ (2 marks)

(ii) $x[n] = \cos(\pi/3)n + \sin(\pi/4)n$ (4 marks)

(b) (i) Find the total energy associated with the exponentially decaying pulse

$$v(t) = Ve^{-t/\tau} \quad \text{for } t \geq 0$$
$$= 0 \quad \text{for } t < 0$$

    (5 marks)

(ii) Calculate the average power of the following signal

$$v(t) = 3$$

    (3 marks)

(c) Sketch and label the following signals

(i) $x(t) = 5\partial(t-1) - \partial(t+1) + 2\partial(t+2)$ (2 marks)
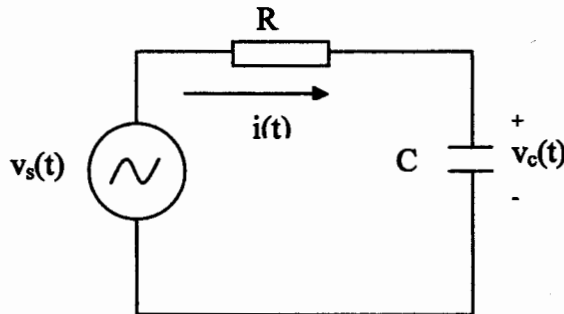
(ii) $x(t) = -3\text{rect}(5x - 3)$ (3 marks)

(d) Given the following RC circuit, derive an equation describing the relationship between the input, $v_s(t)$, and output, $v_c(t)$, voltage signals, and show that the circuit integrates the input signal when the capacitor reactance, X_c, is very small compared to the resistor, R.   (6 marks)
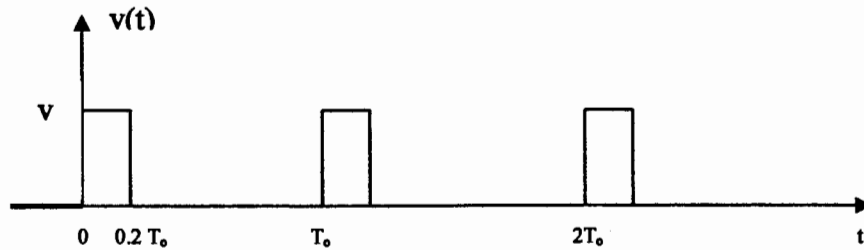
Question 3

(a) Given the following trigonometric Fourier series representation,

$$x(t) = \sum_{k=0}^{\infty} [a_k \cos(k\omega_o t) + b_k \sin(k\omega_o t)]$$

derive the complex exponential Fourier series representation   (5 marks)

(b) Consider the pulse waveform shown below

Using the trigonometric Fourier series analysis, determine the

(i)   average value, a_o (3 marks)
(ii) value of a_k (4 marks)
(iii) value of b_k (4 marks)
(iv) expression for v(t)   (2 marks)

(c) Determine the complex exponential Fourier series representation of the following signal   (7 marks)


∧

Question 4

(a) Determine if the systems producing the following input-output relationship are linear or non-linear

    (i) $y(t) = kx(t)$, k is a constant   (3 marks)

    (ii) $y(t) = x^2(t)$ (3 marks)

(b) Show that the signal $x(t) = e^{j\omega_o t}$ is periodic and that its fundamental period is $2\pi/\omega_o$ (5 marks)

(c) Determine the average value of the following signal   (5 marks)

(d) Determine the average power of the following signal [*Hint:* use Parseval's Theorem]

$$x(t) = 7 - 10\cos(40\pi t - \pi/3) + 4\sin(120\pi t)$$

                                                               (9 marks)

5

Question 5

(a) Consider the following periodic voltage signal.
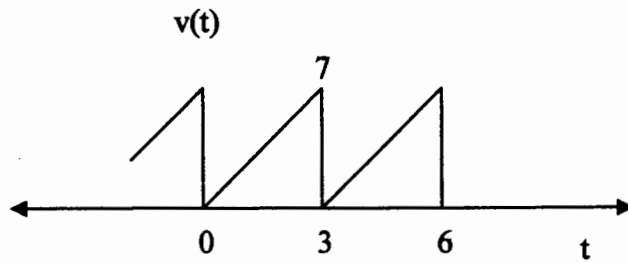
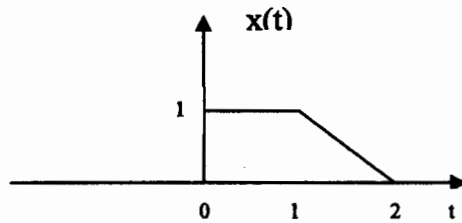Calculate the root mean square (RMS) value   (5 marks)

(b) Determine if the following signals are energy signals, power signals or neither
   (i) x(t) = 4   (1 mark)
   (ii) x(t) = t u(t)   (2 marks)
   (iii) x(t) = e^{-at}u(t)   , a>0   (2 marks)

(c) Sketch and label the even and odd components of the following signal

   x(t) = 2rect(4t + 3)   (5 marks)

(d) With reference to the following signal below, x(t), sketch **x(3t/2 +1)**   (4 marks)

(c)(i) Samples are to be taken from a record of a continuous time signal of duration
       100ms. The signal contains sinusoidal components with the frequencies up to 250
       Hz. Determine the minimum number of samples that would be sufficient to give a
       representation of the signal.   (4 marks)
   (ii) What is aliasing?   (2 marks)