UNIVERSITY OF SWAZILAND

MAIN EXAMINATION 2005/2006

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRONIC ENGINEERING

TITLE OF PAPER:	LINEAR SYSTEMS
COURSE CODE:	E352
TIME ALLOWED:	THREE HOURS

INSTRUCTIONS:

1. **There are SIX questions in this paper. Answer any FOUR questions.**
2. **Questions carry equal marks.**
3. **A table of "Laplace Transforms of Selected Functions" and graphs of "Normalized Response of a Second Order System to a Unit Step Input" are attached.**

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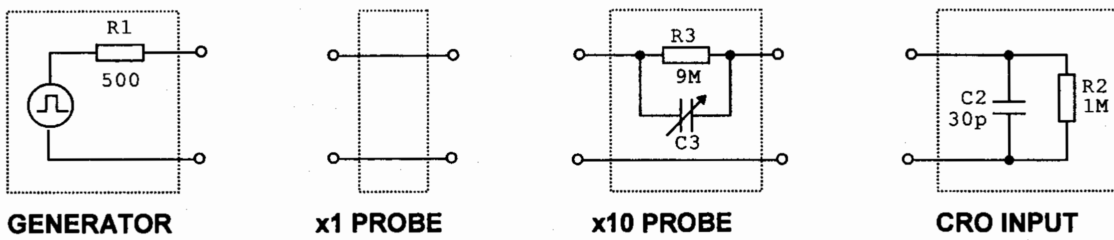
THIS PAPER CONTAINS NINE (9) PAGES INCLUDING THIS PAGE

QUESTION 1 *(25 marks)*

A square wave generator may be connected to an oscilloscope using either a x1 voltage probe or a x10 voltage probe (a x10 probe attenuates the input voltage by a factor of 10 so any voltage readings must be multiplied by 10). The generator, the probes and the oscilloscope input channel are modeled as shown in the following circuit diagrams:



The period of the square wave is assumed to be much greater than the time constants involved. The amplitude of the square wave is adjusted to be 1 volt.

(a) The generator is first connected to the oscilloscope using the x1 probe.

(i) Show that the rising edge of the displayed wave form is of the form

$$v_1(t) = k\left(1-e^{-t/\tau}\right)$$ *(5 marks)*

(ii) Calculate the values of k and τ. *(4 marks)*

(iii) Calculate the rise time of the rising edge of the display. *(2 marks)*

(b) The signal generator is now connected to the oscilloscope using the x10 probe and C_3 is adjusted so that $R_3C_3 = R_2C_2$.

(i) Show that the rising edge of the displayed waveform is also of the form

$$v_2(t) = k'\left(1-e^{-t/\tau'}\right)$$ *(5 marks)*

(ii) Calculate the values of k' and τ'. *(4 marks)*

(iii) Show that at steady state $v_2 / v_1 = \frac{1}{10}$. *(1 mark)*

(iv) Evaluate the rise time of the rising edge. *(2 mark)*

(v) Compare the rise times with the two probes. Which probe would you prefer to use for displaying the waveform and why? *(2 marks)*

QUESTION TWO *(25 marks)*

(a) A waveform is represented by $4\cos\left(3t-\frac{\pi}{3}\right)$.

(i) Obtain the complex amplitude and complex frequency of the waveform. *(5 marks)*

(ii) Show by means of a sketch how the waveform may be generated using a rotating phasor and in your sketch indicate the first time the phasor has a phase of 0 rad and $+\pi/2$ rad. *(4 marks)*

(b) The current *i(t)* and voltage *v(t)* across a linear circuit component are given by

$$i(t) = 6\sin\left(3t+\frac{\pi}{3}\right) \text{ amps}$$

$$v(t) = 10\sin\left(3t+\frac{\pi}{2}\right) \text{ volts}$$

(i) Calculate the impedance of this component. *(4 marks)*

(ii) Obtain an equivalent circuit for the component in terms of lumped R, L and C elements. *(2 marks)*

(c) A waveform $f(t) = 4\cos\left(\frac{\pi}{6}t-\frac{\pi}{4}\right)$ is represented by two contra-rotating phasors, one clockwise (CW) and the other counter clockwise (CCW). Obtain the complex amplitudes of the two phasors. *(10 marks)*

QUESTION THREE ***(25 marks)***

(a) A truck of mass M_1 is coupled to a trailer of mass M_2 using a damper and spring mechanism as shown in Fig. Q3a. The two masses are initially at rest and the wheels may be assumed to be frictionless on the road. In order to move the truck develops a force F(t).

(i) Write down the differential equations for the displacements of the truck and trailer. *(9 marks)*

(ii) Comment on whether this type of coupling is good or not. *(1 mark)*

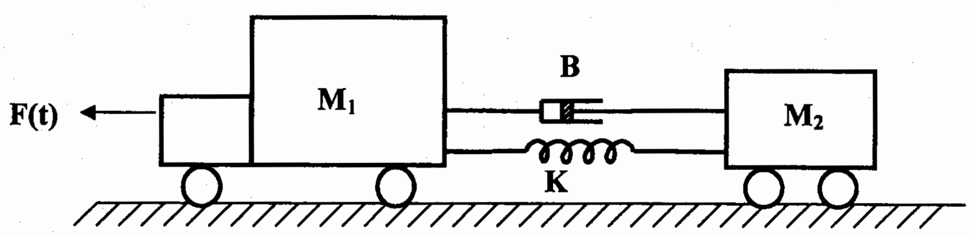


Fig Q3a

(b) The output of a second order system with an input e^{-t} is governed by the differential equation:

$$\frac{d^2y(t)}{dt^2}+2\frac{dy(t)}{dt}+2y(t) = e^{-t}, \quad \text{assume } y(0)=y'(0)=0.$$

(i) Find $y(t)$ using the Laplace Transform method or otherwise. *(11 marks)*

(ii) Sketch $y(t)$ for $0<t<2\pi$ sec. *(4 marks)*

QUESTION FOUR *(25 marks)*

(a) The dynamics of a linear system are represented by the transfer function

$$H(s) = \frac{16}{2s^2 + 4s + 8}$$

A unit step input is applied to the system.

(i) What is the steady state output of the system? *(2 marks)*

(ii) Show that the system oscillations are damped. *(4 marks)*

(iii) Find the frequency of the damped oscillations. *(3 marks)*

(iv) What is the peak output? *(2 marks)*

(v) How long does the system take to settle to within 2% of its steady state output? *(2 marks)*

(vi) Suppose the steady state gain is doubled, what happens to the % overshoot? *(2 marks)*

(b) A block diagram of a closed loop system is shown in Q.4b.

(i) Show that the system has a first order response. *(4 marks)*

(ii) Find the controller gain K_C that will give a closed-loop time constant of 0.05 s. *(3 marks)*

(iii) Find K_C to reduce the steady state error due to a set point input to 10%. *(3 marks)*

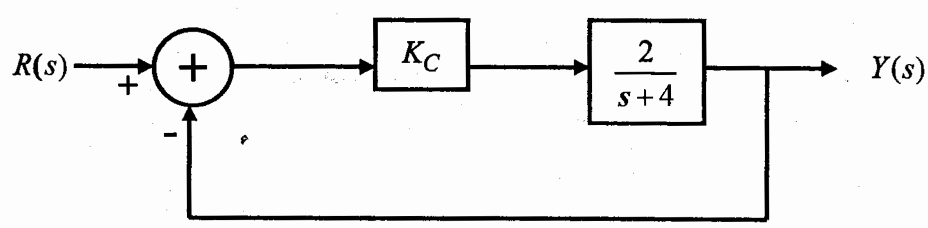


Fig. Q.4b

QUESTION FIVE *(25 marks)*

(a) Express the transfer function of the system shown in the block diagram in Fig. Q.5a as a ratio of polynomials in *s*. *(12 marks)*

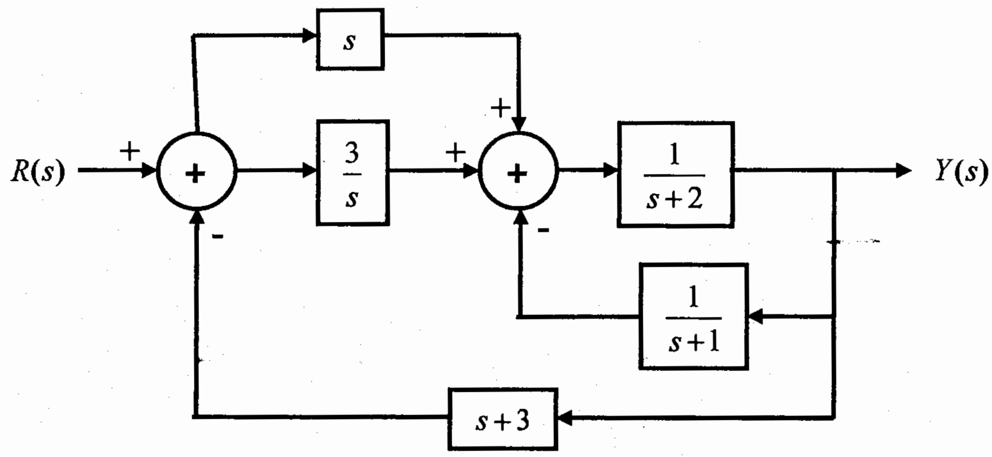


Fig. Q.5a

(b) A system has a transfer function $H(s) = \dfrac{s^2+s+1}{s^3+4s^2+6s+3}$.

(i) Find and sketch its poles and zeros on the s-plane. *(11 marks)*

(ii) Is the system stable? Explain your answer. *(2 marks)*

QUESTION SIX ***(25 marks)***

(a) What are state variables of a system? *(4 marks)*

(b) An electrical circuit is shown in Fig. Q.6b.

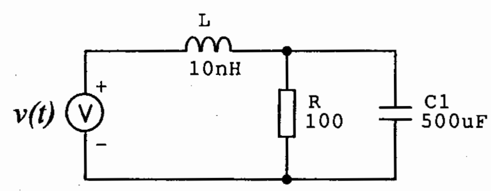


Fig. Q.6b

(i) Choose the state variables for the system, stating reasons for your choice.

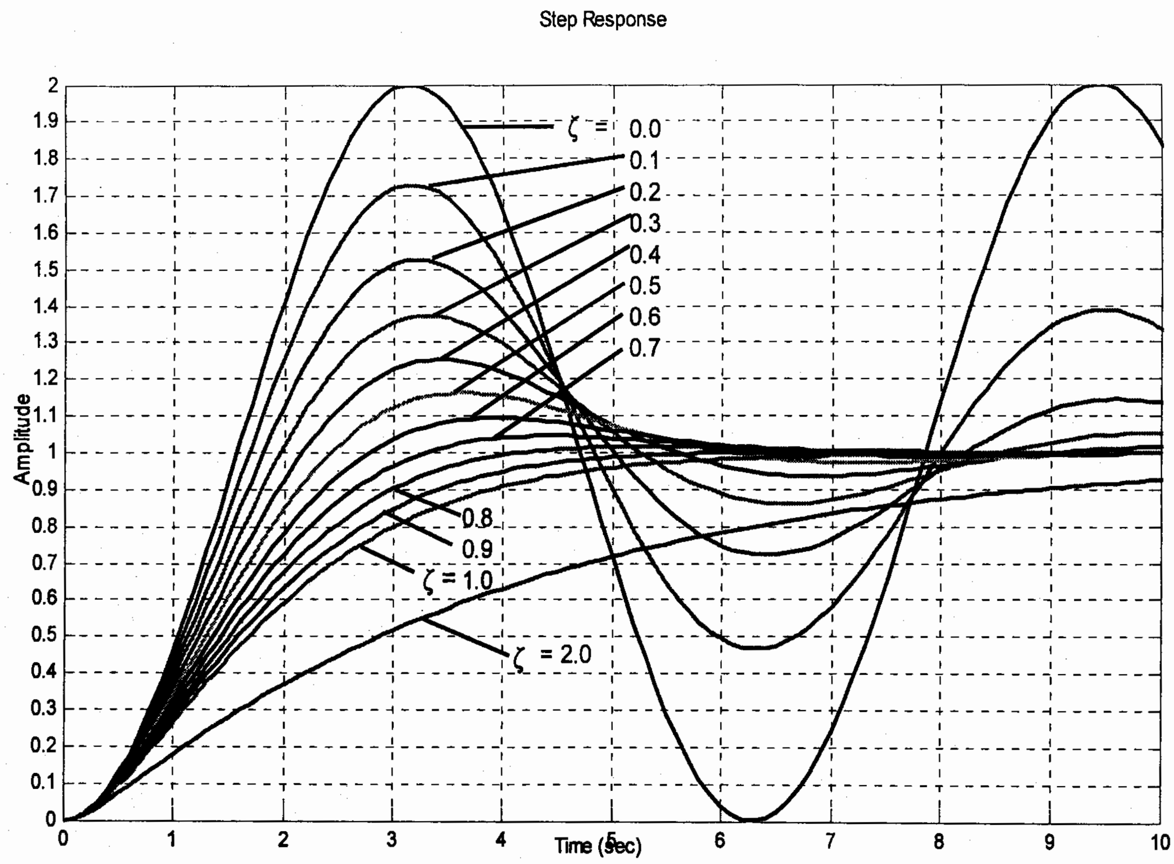
(4 marks)

(ii) Derive its state-space representation if the output is the current in the resistor.

(17 marks)

=====END OF PAPER, ATTACHMENTS FOLLOW=====

NORMALIZED RESPONSE OF A SECOND ORDER SYSTEM TO A UNIT STEP INPUT



STANDARD FORM OF A SECOND-ORDER SYSTEM

$$\frac{d^2y}{dt^2} + 2\zeta\omega_n \frac{dy}{dt} + \omega_n^2 y = k\omega_n^2$$

TABLE OF LAPLACE TRANSFORMS OF SELECTED FUNCTIONS

$f(t),\ t>0$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
t	$\frac{1}{s^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{at} f(t)$	$F(s-a)$
$\frac{t}{2\omega} \sin \omega t$	$\frac{s}{\left(s^2+\omega^2\right)^2}$
$e^{-\alpha t} \sin \omega t$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
$e^{-\alpha t} \cos \omega t$	$\frac{(s+\alpha)}{(s+\alpha)^2+\omega^2}$