UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION 2005/2006

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRONIC ENGINEERING

TITLE OF PAPER:	LINEAR SYSTEMS
COURSE CODE:	E352
TIME ALLOWED:	THREE HOURS

INSTRUCTIONS:

1. **There are FIVE questions in this paper. Answer Question 1 and any other TWO questions.**
2. **Question one carries 50 marks while the other questions carry 25 marks each.**
3. **A table of "Laplace Transforms of Selected Functions" and graphs of "Normalized Response of a Second Order System to a Unit Step Input" are attached.**

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR

THIS PAPER CONTAINS NINE (9) PAGES INCLUDING THIS PAGE

QUESTION ONE (COMPULSORY) ***(50 marks)***

(a) Write down a mathematical expression for an oscillatory signal whose amplitude increases linearly with time. *(3 marks)*

(b) Write down an expression for a waveform whose phase is -60° at t=0 and whose amplitude decays exponentially with a time-constant of 1.5 s. *(4 marks)*

(c) Write down the transfer function of a first-order system with a d.c. gain of 2 and a time-constant of 0.7 s. *(4 marks)*

(d) Obtain a differential equation relating the output voltage $v_o(t)$ to the input voltage $v_i(t)$ in the circuit shown in Fig. 1.1. No other variables should appear in your expression, *(6 marks)*

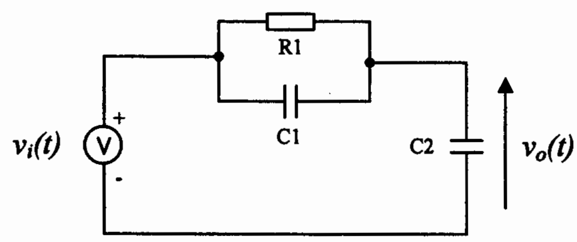


Fig. 1.1

(e) The output *x(t)* of a linear system with an input impulse input $\delta(t)$ is described by the equation:

$$\frac{d^2x}{dt^2}+8\frac{dx}{dt}+15x = \delta(t)$$

with the initial conditions $\dot{x}(0) = x(0) = 0$. Solve for *x(t)*. *(6 marks)*

(f) A causal signal *f(t)* whose Laplace Transform is *F(s)* is passed through a pure delay system with time delay *T*. The signal amplitude is not affected by the system. Write down an expression for the output signal in the time domain and derive the Laplace Transform of the output signal. *(5 marks)*

Question One (continued)

(g) The input voltage v_{in} volts and angular speed $\Omega(t)$ of a motor are related by the equation:

$$\frac{d\Omega}{dt} + 0.2\Omega = v_{in}$$

The motor is initially at rest and a 50 V step voltage is applied. Find how long the motor takes to reach 90% of its final speed and in addition find its final speed in revolutions per minute. *(9 marks)*

(h) The pole zero diagram of a system is shown in Fig.1.2. Find the transfer function of the system. *(4 marks)*

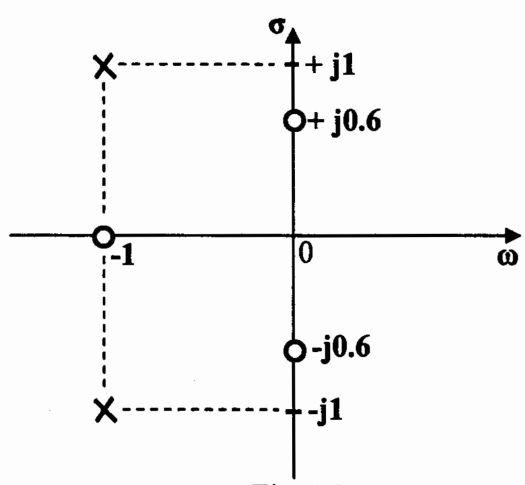


Fig. 1.2

(i) Draw a block diagram of a system represented by the following equations:

$$X_1 - \frac{1}{20}X_4 = X_3 \text{ and } 40X_3 = X_4 \text{ and } X_2 = 10X_3 + 30X_4$$

where X_1 is the input and X_2 is the output.

Reduce your block diagram to get the gain, X_2/X_1, of the system. *(9 marks)*

QUESTION TWO ***(25 marks)***

(a) The voltage $v(t)$ across, and the current $i(t)$ in a circuit component are given by:

$$v(t) = 3e^{-5t}\cos\left(4t+\frac{\pi}{3}\right)$$

$$i(t) = 6e^{-5t}\cos\left(4t-\frac{\pi}{5}\right)$$

Find the impedance of the component and find its equivalent circuit in terms of lumped RLC elements. *(11 marks)*

(b) Find the poles and zeros of the transfer function of the circuit shown in Fig. 2.1.

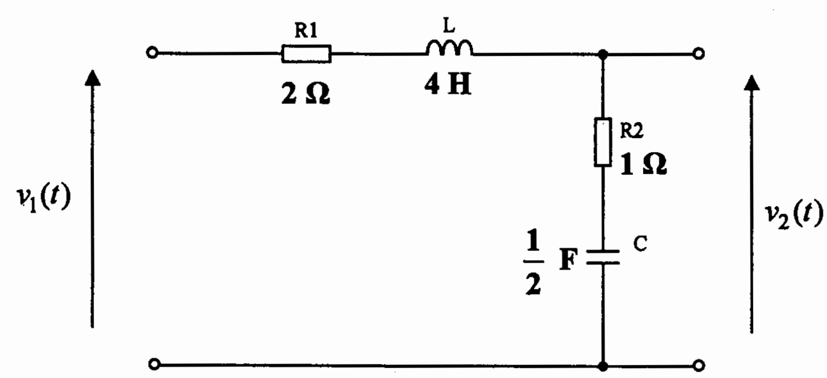


Fig. 2.1

QUESTION THREE ***(25 marks)***

Fig. 3.1 represents a "times 10" oscilloscope probe. The capacitor C_1 is adjusted so that $R_1C_1 = R_2C_2$. V_1 is the voltage being measured and V_o is the voltage appearing at the oscilloscope input channel.

(a) Obtain the transfer function $V_o(s)/V_1(s)$ of the probe, and show that the probe behaves like a first-order system. *(15 marks)*

(b) Find the time constant. *(5 marks)*

(c) Find the steady state gain and show that the displayed voltage V_o must be multiplied by 10 to get the value of the voltage being measured V_1.

(5 marks)

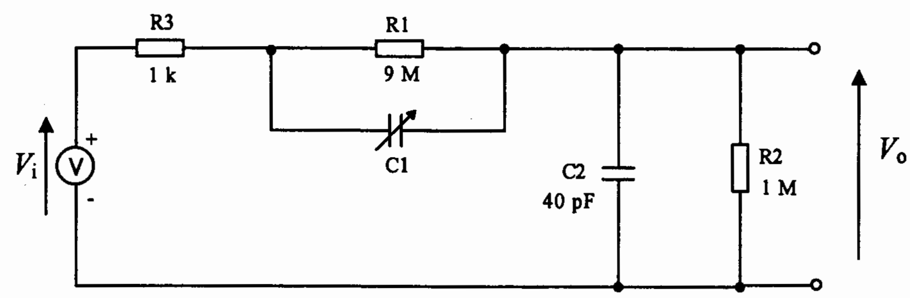


Fig.3.1

QUESTION FOUR ***(25 marks)***

In the circuit shown in Fig. 4.1 the input is the voltage $v_1(t)$ and the output is the current $i_2(t)$. Choose state variables, and derive the state equation and the output equation, expressing your result in matrix form. *(25 marks)*

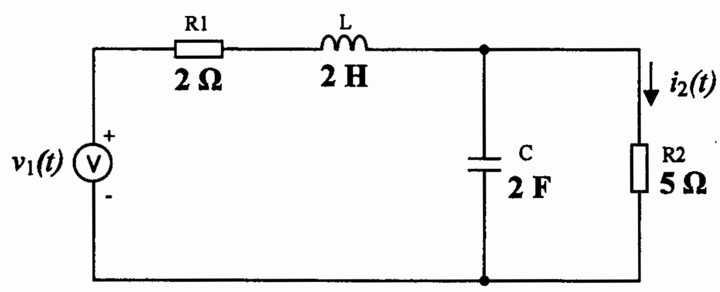


Fig. 4.1

QUESTION FIVE ***(25 marks)***

A control system is represented by the block diagram in Fig.5.1

If the input is a step command of amplitude A,

(a) Find the steady state value and the steady state error at the output of the system. *(9 marks)*

(b) Show that the system is oscillatory with damped oscillations. *(4 marks)*

(c) Find the frequency of the damped oscillations. *(4 marks)*

(d) Find the Percentage peak overshoot. *(4 marks)*

(e) Find the settling time to within 2% of the steady state value. *(4 marks)*

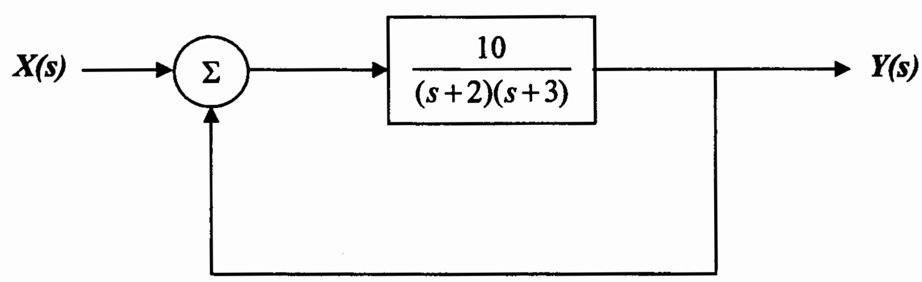
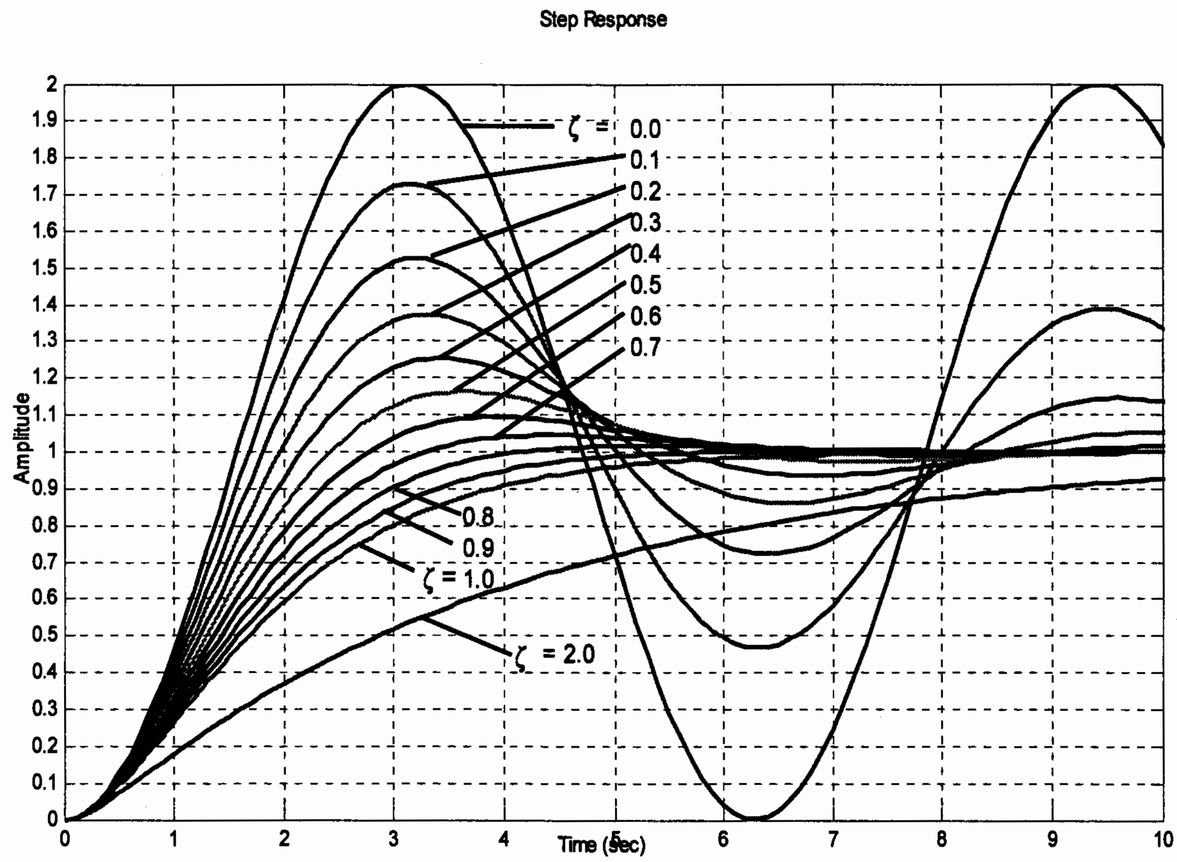


Fig. 5.1

=====*END OF PAPER, ATTACHMENTS FOLLOW*=====

NORMALIZED RESPONSE OF A SECOND ORDER SYSTEM TO A UNIT STEP INPUT



STANDARD FORM OF A SECOND-ORDER SYSTEM

$$\frac{d^2y}{dt^2} + 2\zeta\omega_n\frac{dy}{dt} + \omega_n^2 y = k\omega_n^2$$

TABLE OF LAPLACE TRANSFORMS OF SELECTED FUNCTIONS

$f(t),\ t>0$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
t	$\frac{1}{s^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{at} f(t)$	$F(s-a)$
$\frac{t}{2\omega}\sin \omega t$	$\frac{s}{\left(s^2+\omega^2\right)^2}$
$e^{-\alpha t}\sin \omega t$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
$e^{-\alpha t}\cos \omega t$	$\frac{(s+\alpha)}{(s+\alpha)^2+\omega^2}$