UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRONIC ENGINEERING

SUPPLEMENTARY EXAMINATION 2006

TITLE OF PAPER	:	MATHEMATICAL METHODS I (PAPER ONE)
COURSE NUMBER	:	E370(I)
TIME ALLOWED	:	THREE HOURS
INSTRUCTIONS	:	ANSWER ANY FOUR OUT OF FIVE QUESTIONS. EACH QUESTION CARRIES 25 MARKS. MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.

THIS PAPER HAS SEVEN PAGES, INCLUDING THIS PAGE.

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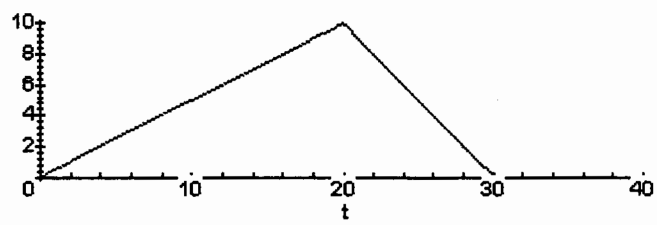
Question one

(a) Given the following function $f(t)$ as



(i) express $f(t)$ in terms of Heaviside functions and then plot it for $t = 0$ *to* 40 to reproduce the given graph, (5 marks)

(ii) find the Laplace transform of $f(t)$ by integration, (4 marks)

(iii) apply the first shifting theorem to find the Laplace transform of $e^{7t} f(t)$. (2 marks)

Question one (continued)

(b) **(i)** **Find the inverse Laplace transforms of both** $F(s) = \dfrac{2+8s}{(s+3)(s+1)(s-2)}$

and $G(s) = \dfrac{s^2 - 5s - 8}{s^4 - 6s^2}$, **(4 marks)**

(ii) **apply the second shifting theorem to find the inverse Laplace transform of**

$e^{-9s}F(s)$, **(2 marks)**

(iii) **apply the convolution theorem to find the inverse Laplace transform of**

$F(s)G(s)$ **and plot it for** $t = 0$ *to* 10 . **(8 marks)**

Question two

Given the following differential equation $y'' - 5y' + 8y = r(t)$ **where**

$r(t) = 5(u(t-5) - u(t-10))$ **and its initial conditions** $y(0) = 6$ *and* $y'(0) = 0$

(a) **find the Laplace transform of** $r(t)$ **, namely** $R(s)$, **(2 marks)**

(b) **express the Laplace transform of** $y(t)$ **, namely** $Y(s)$, **as**

$Y(s) = F(s)R(s) + G(s)$ **, find the expressions of** $F(s)$ **and** $G(s)$,

(6 marks)

(c) **find the inverse Laplace transforms of** $F(s)$ **and** $G(s)$ **, namely** $f(t)$ **and** $g(t)$. **(2 marks)**

(d) **find the convolution of** $f(t) * r(t)$ **in** $0 \le t < 5$, $5 \le t < 10$ **and** $10 < t$ **explicitly, (10 marks)**

(e) **plot** $y(t)$ **, which is** $f(t) * r(t) + g(t)$ **, for** $t = 0$ *to* 20 . **(5 marks)**

Question three

(a) **Given the following systems of linear equations :**

$$\begin{cases} 4x - 8y + 3z = 16 \\ -x + 4y - 5z = -21 \\ 3x - 6y + z = 7 \end{cases}$$

(i) **use** *linsolve* **command to find the solutions ,** **(3 marks)**

(ii) **find the solutions by using the method of Gauss elimination, (use** *addrow* **command successively and then use** *backsub* **command to get the solutions)** **(6 marks)**

(ii) **find the solutions by using the Cramer's rule.** **(6 marks)**

(b) **Given the following matrix** $B = \begin{pmatrix} 4 & 1 & 0 \\ 5 & -3 & 1 \\ -9 & 4 & -1 \end{pmatrix}$ **,**

(i) **find its inverse** B^{-1} **by the Gauss-Jordan elimination method (use** *addrow* **command in succession and also use** *mulrow* **command).**

(8 marks)

(ii) **use** *inverse* **command to find** B^{-1} **and compare it with that obtained in (b)(i) .** **(2 marks)**

Question four

(a) Given the following system of differential equations :

$$\begin{cases} \dfrac{d\,x_1(t)}{dt} = -4x_1(t) + 2x_2(t) \\ \dfrac{d\,x_2(t)}{dt} = x_1(t) - 3x_2(t) \end{cases}$$

(i) set $x_1(t) = X_1 e^{\lambda t}$ *and* $x_2(t) = X_2 e^{\lambda t}$, substituting them into the above equations and deduce the following matrix equation $Ay = \lambda I y$

where $A = \begin{pmatrix} -4 & 2 \\ 1 & -3 \end{pmatrix}$, $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ *and* $y = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$ (4 marks)

(ii) find the eigenvalues of λ by using $\det(A - \lambda I) = 0$, (4 marks)

(iii) use *eigenvects* commend to find the eigenvalues and eigenvectors of A . Compare the eigenvalues found here to those obtained in (ii) and make brief comments . (3 marks)

(iv) write down the general solution of the x_1 *and* x_2 in terms of their eigenvalues and eigenvectors . Determine the values of the arbitrary constants in the general solutions if $x_1(0) = 7$ *and* $x_2(0) = 5$.

(6 marks)

(b) For a normal distribution $f(x)$ with the mean value of 25 and the standard deviation of 15 ,

(i) plot $f(x)$ for $x = 0$ *to* 40 , (3 marks)

(ii) find its corresponding cumulative distribution function $g(x)$ and use it to calculate the values of the probabilities of $P(x > 7)$ and $P(9 < x < 19)$

(5 marks)

Question five

(a) **Given the following differential equation** $4x\frac{d^2 y(x)}{dx^2} + 2\frac{d\,y(x)}{dx} + y(x) = 0$ **,**

using the power series method , i.e., set $y(x) = \sum_{n=0}^{\infty} a_n\, x^{n+s}$ **with** $a_0 \neq 0$ **and substituting it back to the given differential equation ,**

(i) **requiring the coefficients of the lowest power terms for** x **, i.e.,** x^{s-1} **, to be zero and thus write down the indicial equations . From the equation find the values of** s **(possibly also the value of** a_1 **) ,** **(6 marks)**

(ii) **requiring the coefficients of all the rest power terms for** x **, i.e.,** x^{s+n} **with** $n = 0, 1, 2, 3, \cdots$ **, to be zero and find the recurrence relation,**

(4 marks)

(iii) **using the recurrence relation in (a)(ii) , find the values of** a_1 **,** a_2 **&** a_3 **if** $a_0 = 1$ **for each value of** s **found in (a)(i) .** **(8 marks)**

(b) **If the right hand side of the differential equation in (a) is** $5x^2 - 3$ **instead of zero , then set a particular solution for it as** $y_p(x) = k_1\, x^2 + k_2\, x + k_3$ **and find the values of** k_1 **,** k_2 **and** k_3 **. Write down the general solution for (b) .**

(7 marks)