**UNIVERSITY OF SWAZILAND**
**FACULTY OF SCIENCE**
**DEPARTMENT OF ELECTRONIC ENGINEERING**

**MAIN EXAMINATION 2006**

**TITLE OF PAPER : SIGNALS II**

**COURSE NUMBER : E462**

**TIME ALLOWED : THREE (3) HOURS**

**INSTRUCTIONS : ANSWER ANY FOUR OUT OF THE FIVE QUESTIONS**

**EACH QUESTION CARRIES 25 MARKS**

**MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN**

**THIS PAPER CONTAINS SEVEN (7) PAGES INCLUDING THIS PAGE**

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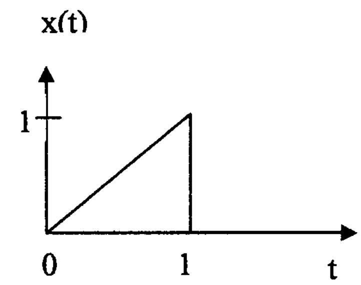
# Question 1

(a) From the definition of the Fourier Transform, prove that

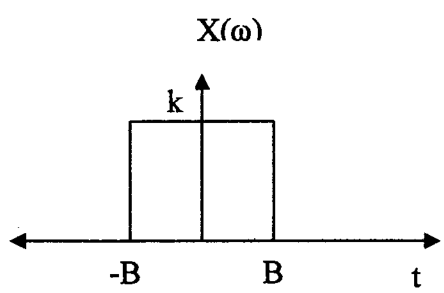
$$x(t - t_o) \Longleftrightarrow e^{-j\omega t_o} X(\omega)$$ (3 marks)

(b) (i) Find the Fourier Transform of the following waveform at frequency, f = 2 Hz.




(10 marks)

(ii) Find the Inverse Fourier Transform of the following signal




(5 marks)

(c) Given the Fourier Transform pair

$$e^{-|t|} \Longleftrightarrow \frac{2}{1 + \omega^2}$$

determine the Fourier Transform of $te^{-|t|}$ (7 marks)

[*hint:* use the appropriate Fourier Transform property in **Table 1** on the back page]

## Question 2

(a) Given that a DSP system is described by a unit sample response, h[n] = [3, 1, 0.5], determine the output sequence of the system in response to the digital input sequence x[n] = [2, 1, 0, -1, -2]

(10 marks)

(b) (i) What is correlation? Give one typical application example where correlation is used. (3 marks)

(ii) Perform the cross-correlation, $C_{xh}(p)$, of the two sequences namely x[n] = [2, 1, 3, 0] and h[n] = [3, 2, 4, 3]

(7 marks)

(c) Verify the commutative property of the convolution integral, that is,

$$x(t) * h(t) = h(t) * x(t),$$ (5 marks)

where x(t) is the input and h(t) is the impulse response of the system.

## Question 3

(a) An N-sample signal x[n] has the Discrete Fourier Transform (DFT) X[k]. Write down the expression for the DFT of the signal 2x[n] + x[n+1]

(2 marks)

(b) Evaluate the 4-point DFT for the signal **x(t) = sin(2π1000t) volts**, at a sampling frequency of 4 kHz

(10 marks)

(c) Plot the magnitude spectrum of the result in (b) (5 marks)

(d) Verify that in the discrete frequency space the spectral coefficient **X[3]** = **X[7]** in (b) above

(3 marks)

(e) Using the Inverse Discrete Fourier Transform (IDFT), verify that in the discrete time space **x[1]** = **x[5]** in (b) above

(5 marks)

## Question 4

(a) A box contains 30 resistors: 15 of the resistors have nominal values of 1.0 KΩ, 10 have nominal values of 4.7 KΩ and 5 have nominal values of 10 KΩ ; 3 resistors are taken at random and connected in series. What is the probability that the 3-resistor combination will have a nominal resistance of

(i) 3 KΩ (4 marks)

(ii) 15.7 KΩ (4 marks)

(iii) 19.4 KΩ (4 marks)

(b) Consider transmitting a three-digit message over a noisy channel having error probability P(E) =2/5 per digit. Assuming statistical independence, calculate the probability of receiving

(i) a correct digit (2 marks)

(ii) a message with one error (4 marks)

(c) Given that the sampling period T = 1s, determine the first four values and the final value of the input sequence x[n] for

$$X(z) = \frac{z(1 - e^{-T})}{(z - 1)(z - e^{-T})}$$

(7 marks)

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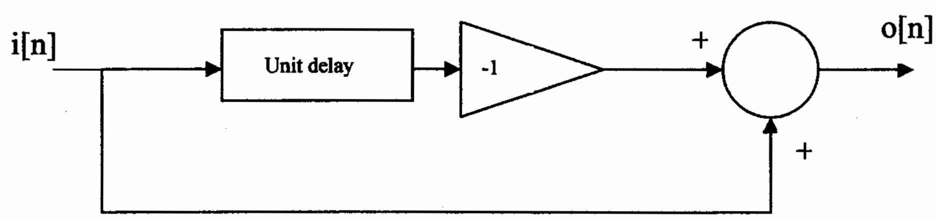
## Question 5

(a) Define the term 'filtering' (2 marks)

(b) Explain how Butterworth and Chebyshev filters approximate an ideal, 'rectangular', response characteristic

(4 marks)

(c) A first order digital filter has the structure shown below.




(i) Find its transfer function, H(z) (4 marks)

By first finding the z-transform of the following input sequences, find the corresponding output sequences, o[n].

(ii) i[n] = {1, 0, 0 ,0 ,0, ...} (4 marks)

(iii) i[n] = {1, 1, 1, 1,...} (4 marks)

(d) Find the z-transform of the following sequence and simplify your answer.

$x[n] = \cos(\pi n/4)$ (7 marks)

Table 1. Properties of the Fourier Transform

| Property | Signal | Fourier transform |
|---|---|---|
| | $x(t)$ | $X(\omega)$ |
| | $x_1(t)$ | $X_1(\omega)$ |
| | $x_2(t)$ | $X_2(\omega)$ |
| Linearity | $a_1x_1(t)+a_2x_2(t)$ | $a_1X_1(\omega)+a_2X_2(\omega)$ |
| Time shifting | $x(t-t_0)$ | $e^{-j\omega t_0}X(\omega)$ |
| Frequency shifting | $e^{j\omega_0 t}x(t)$ | $X(\omega-\omega_0)$ |
| Time scaling | $x(at)$ | $\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$ |
| Time reversal | $x(-t)$ | $X(-\omega)$ |
| Duality | $X(t)$ | $2\pi x(-\omega)$ |
| Time differentiation | $\frac{dx(t)}{dt}$ | $j\omega X(\omega)$ |
| Frequency differentiation | $(-jt)x(t)$ | $\frac{dX(\omega)}{d\omega}$ |
| Integration | $\int_{-\infty}^{t} x(\tau)\,d\tau$ | $\pi X(0)\delta(\omega)+\frac{1}{j\omega}X(\omega)$ |
| Convolution | $x_1(t)*x_2(t)$ | $X_1(\omega)X_2(\omega)$ |
| Multiplication | $x_1(t)x_2(t)$ | $\frac{1}{2\pi}X_1(\omega)*X_2(\omega)$ |
| Real signal | $x(t)=x_e(t)+x_o(t)$ | $X(\omega)=A(\omega)+jB(\omega)$ |
| | | $X(-\omega)=X^*(\omega)$ |
| Even component | $x_e(t)$ | $\mathrm{Re}\{X(\omega)\}=A(\omega)$ |
| Odd component | $x_o(t)$ | $j\,\mathrm{Im}\{X(\omega)\}=jB(\omega)$ |
| Parseval's relations | | |

$$\int_{-\infty}^{\infty} x_1(\lambda)X_2(\lambda)\,d\lambda=\int_{-\infty}^{\infty} X_1(\lambda)x_2(\lambda)\,d\lambda$$

$$\int_{-\infty}^{\infty} x_1(t)x_2(t)\,dt=\frac{1}{2\pi}\int_{-\infty}^{\infty} X_1(\omega)X_2(-\omega)\,d\omega$$

$$\int_{-\infty}^{\infty} |x(t)|^2\,dt=\frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\,d\omega$$

Table 2 Some Common z-Transform Pairs

| $x[n]$ | $X(z)$ | ROC |
|---|---|---|
| $\delta[n]$ | $1$ | All $z$ |
| $u[n]$ | $\frac{1}{1-z^{-1}},\frac{z}{z-1}$ | $\lvert z\rvert>1$ |
| $-u[-n-1]$ | $\frac{1}{1-z^{-1}},\frac{z}{z-1}$ | $\lvert z\rvert<1$ |
| $\delta[n-m]$ | $z^{-m}$ | All $z$ except 0 if $(m>0)$ or $\infty$ if $(m<0)$ |
| $a^n u[n]$ | $\frac{1}{1-az^{-1}},\frac{z}{z-a}$ | $\lvert z\rvert>\lvert a\rvert$ |
| $-a^n u[-n-1]$ | $\frac{1}{1-az^{-1}},\frac{z}{z-a}$ | $\lvert z\rvert<\lvert a\rvert$ |
| $na^n u[n]$ | $\frac{az^{-1}}{(1-az^{-1})^2},\frac{az}{(z-a)^2}$ | $\lvert z\rvert>\lvert a\rvert$ |
| $-na^n u[-n-1]$ | $\frac{az^{-1}}{(1-az^{-1})^2},\frac{az}{(z-a)^2}$ | $\lvert z\rvert<\lvert a\rvert$ |
| $(n+1)a^n u[n]$ | $\frac{1}{(1-az^{-1})^2},\left[\frac{z}{z-a}\right]^2$ | $\lvert z\rvert>\lvert a\rvert$ |
| $(\cos\Omega_0 n)u[n]$ | $\frac{z^2-(\cos\Omega_0)z}{z^2-(2\cos\Omega_0)z+1}$ | $\lvert z\rvert>1$ |
| $(\sin\Omega_0 n)u[n]$ | $\frac{(\sin\Omega_0)z}{z^2-(2\cos\Omega_0)z+1}$ | $\lvert z\rvert>1$ |
| $(r^n\cos\Omega_0 n)u[n]$ | $\frac{z^2-(r\cos\Omega_0)z}{z^2-(2r\cos\Omega_0)z+r^2}$ | $\lvert z\rvert>r$ |
| $(r^n\sin\Omega_0 n)u[n]$ | $\frac{(r\sin\Omega_0)z}{z^2-(2r\cos\Omega_0)z+r^2}$ | $\lvert z\rvert>r$ |
| $\begin{cases}a^n & 0\le n\le N-1\\ 0 & \text{otherwise}\end{cases}$ | $\frac{1-a^N z^{-N}}{1-az^{-1}}$ | $\lvert z\rvert>0$ |