UNIVERSITY OF SWAZILAND
FACULTY OF SCIENCE

DEPARTMENT OF ELECTRONIC ENGINEERING
MAIN EXAMINATION 2005 / 2006

TITLE OF PAPER : DIGITAL COMMUNICATIONS

COURSE NUMBER : E530

TIME ALLOWED : THREE HOURS

INSTRUCTIONS : READ EACH QUESTION CAREFULLY
ANSWER ANY **FOUR** OUT OF **FIVE** QUESTIONS. EACH QUESTION CARRIES **25 MARKS**. MARKS FOR EACH SECTION ARE SHOWN ON THE RIGHT-HAND MARGIN.

THIS PAPER HAS 6 PAGES INCLUDING THIS PAGE.

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USEFUL INFORMATION

$$\text{Cos}(A \pm B) = \text{Cos}\,A\,\text{Cos}\,B \mp \text{Sin}\,A\,\text{Sin}\,B$$
$$\text{Sin}(A \pm B) = \text{Sin}\,A\,\text{Cos}\,B \pm \text{Cos}\,A\,\text{Sin}\,B.$$

$$\text{Sin}\,A\,\text{Sin}\,B = \tfrac{1}{2}[\text{Cos}(A - B) - \text{Cos}(A + B)]$$
$$\text{Cos}\,A\,\text{Cos}\,B = \tfrac{1}{2}[\text{Cos}(A + B) + \text{Cos}(A - B)]$$
$$\text{Sin}\,A\,\text{Cos}\,B = \tfrac{1}{2}[\text{Sin}(A + B) + \text{Sin}(A - B)]$$

$$\text{Cos}^2A = \tfrac{1}{2}[1 + \text{Cos}\,2A]$$
$$\text{Sin}^2A = \tfrac{1}{2}[1 - \text{Cos}\,2A]$$

Table 1 Values for Q(x)

x	10log x	Q(x)	x	10log x	Q(x)	x	10log x	Q(x)
3.00	4.77	1.35 E-03	4.00	6.02	3.17 E-05	5.00	6.99	2.87 E-07
3.05	4.84	1.14 E-03	4.05	6.07	2.56 E-05	5.05	7.03	2.21 E-07
3.10	4.91	9.68 E-04	4.10	6.13	2.07 E-05	5.10	7.08	1.70 E-07
3.15	4.98	8.16 E-04	4.15	6.18	1.66 E-05	5.15	7.12	1.30 E-07
3.20	5.50	6.87 E-04	4.20	6.23	1.30 E-05	5.20	7.16	9.96 E-08
3.25	5.12	5.77 E-04	4.25	6.28	1.07 E-05	5.25	7.20	7.61 E-08
3.30	5.19	4.83 E-04	4.30	6.33	8.54 E-06	5.30	7.24	5.79 E-08
3.35	5.25	4.04 E-04	4.35	6.38	6.81 E-06	5.35	7.28	4.40 E-08
3.40	5.31	3.37 E-04	4.40	6.43	5.41 E-06	5.40	7.32	3.33 E-08
3.45	5.38	2.80 E-04	4.45	6.48	4.29 E-06	5.45	7.36	2.52 E-08
3.50	5.44	2.33 E-04	4.50	6.53	3.40 E-06	5.50	7.40	1.90 E-08
3.55	5.50	1.93 E-04	4.55	6.58	2.68 E-06	5.55	7.44	1.43 E-08
3.60	5.56	1.59 E-04	4.60	6.63	2.11 E-06	5.60	7.48	1.07 E-08
3.65	5.62	1.31 E-04	4.65	6.67	1.66 E-06	5.65	7.52	8.03 E-09
3.70	5.68	1.08 E-04	4.70	6.72	1.30 E-06	5.70	7.56	6.00 E-09
3.75	5.74	8.84 E-05	4.75	6.77	1.02 E-06	5.75	7.60	4.47 E-09
3.80	5.80	7.23 E-05	4.80	6.81	7.93 E-07	5.80	7.63	3.32 E-09
3.85	5.85	5.91 E-05	4.85	6.86	6.17 E-07	5.85	7.67	2.46 E-09
3.90	5.91	4.81 E-05	4.90	6.90	4.79 E-07	5.90	7.71	1.82 E-09
3.95	5.97	3.91 E-05	4.95	6.95	3.71 E-07	5.95	7.75	1.34 E-09

QUESTION 1

(a)A binary channel of capacity 1 bit per symbol can transmit a binary 0 or 1 at a rate of 2 symbols per second with negligible error. A discrete binary source produces symbols at a rate of 2.5 symbols per second. It has 2 possible outputs A and B, occurring with probabilities of 0.9 and 0.1 respectively.
Explain, showing all calculations, how the discrete binary source output can be represented to enable transmission through the available channel. (10 marks)

(b) Compute:

(i) the average codeword length per original source symbol if second - order extension of the original source were to be used in the encoding process, (10 marks)

(ii) the symbol rate at the encoder output and (2 marks)

(iii) the efficiency of the code. (3 marks)

QUESTION 2

(a) A convolutional code is defined by the following generator polynomials:

$$g_1(X) = 1 + X^2$$
$$g_2(X) = 1 + X$$

Give the circuit diagram of the encoder and determine its impulse response. (6 marks)

(b) Assume the received data sequence 1010001010 is for the encoder

of (a). Using the Viterbi Algorithm,

(i) Identify the errors. (15 marks)

(ii) Derive the original information sequence. (2 marks)

(iii) Compute the effective code rate. (2 marks)

QUESTION 3

(a) A rate of 10^6 bits per second is used for transmission of binary data over a microwave link. The power spectral density at the coherent binary FSK receiver input is 10^{-10} W/Hz. Compute the average carrier power required to maintain an average probability of symbol error Pe $\leq 10^{-4}$.
(7 marks)

(b) A coherent binary phase-shift keying (PSK) system has a transmit signal as follows:

$$S(t) = \pm\sqrt{\frac{2E_b}{T_b}}\,Cos(\omega_c t) \qquad \text{for}$$

$$0 \leq t < T\,.$$

The plus sign is for message m_1 and minus sign for message m_2. E_b is the transmitted signal energy per bit, f_c is the carrier frequency and T_b is the bit period.

(i) Determine the coordinates of the message points for the given PSK signal. (10 marks)

(ii) Present the corresponding signal space diagram. (3 marks)

(iii) Assuming an additive white Gaussian noise (AWGN) channel,

how can you decide on which one of the two signals was sent? (5 marks)

QUESTION 4

(a) (i) Derive a simple decision rule which can be used for recovering the original data from a precoded modified duobinary received sequence. (10 marks)

(ii) Present a circuit design which can be used in the implementation of the decision rule of (i). (4 marks)

(b) A 20 MHz carrier signal is used in Amplitude Shift Keying (ASK) for the transmission of binary data at 20kbps. The additive noise power is 10^{-12} W/Hz and the received carrier amplitude is 10^{-3} V.

(i) Design a coherent detector for the received signal and (9 marks)

(ii) compute the probability of error for the detector. (2 marks)

QUESTION 5

(a)You are given a set of two finite-energy signals

$$S_1(t) = V_c Cos(\omega t)$$
$$S_2(t) = V_c Sin(\omega t)$$

These are defined on the same interval $0 \leq t \leq Ts$.

Determine the number of orthonormal basis functions for the signal space spanned by these signals. (15 marks)

(b) Present a geometrical representation of the signals given that Vc = 1 V and Ts = 8×10^{-6} sec. (10 marks)

QUESTION 6

Consider a systematic linear block code with a minimum distance d_{min} which can correct up to (d_{min} - 1)/2 errors and detect up to d_{min} - 1 errors in each codeword.

(a) Design a linear block code with d_{min} = 3 and a message block size of 11. (11 marks)

(b) Use a worked example to show the number of errors which can be corrected by such a code. (10 marks)

(c) Show that the corrected code vector of (b) is a valid codeword. (4 marks)
