UNIVERSITY OF SWAZILAND
FACULTY OF SCIENCE
DEPARTMENT OF ELECTRONIC ENGINEERING

MAIN EXAMINATION MAY 2007

TITLE OF PAPER: **LINEAR SYSTEMS**

COURSE CODE: **E352**

TIME ALLOWED: **THREE** HOURS

INSTRUCTIONS:

1. Answer question **one** and any other **three** questions.
2. Question one carries 40 marks.
3. Questions 2, 3, 4, and 5 carry 20 marks each.
4. Marks for different sections are shown in the right-hand margin

This paper has 7 pages including this page.

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Question 1

(a) For a diode, the equation relating current I_D and the potential difference V_D may be written as

$$\log I_D = \log I_s + 0.43\frac{V_D}{\eta V_T}$$

where I_S and η are constants,

V_D is the input and I_D is the output.

Find out whether this equation represent the input-output relation of a linear system. (5 marks)

(b) A thermistor has its response to temperature represented by $R = R_0 e^{-0.1T}$

where R = resistance, R_0 = 10 k Ω, and T= temperature in degrees Celsius,

Find the linear model for the thermistor suitable for a small range of variation of temperature when operating at T=20^{0}C (6 marks)

(c) Obtain a differential equation relating the output current $i_0(t)$ to the input voltage $v_i(t)$ in the circuit shown in Figure 1(c). No other variable should appear in your expression. (6 marks)

(d) The output of a linear system for a spacecraft platform is governed by the following equations:

$$\frac{d^2p}{dt^2} + 2\frac{dp}{dt} + 4p = \theta$$

$$v_1 = r - p$$

$$\frac{d\theta}{dt} = 0.6v_2$$

$$v_2 = 7v_1$$

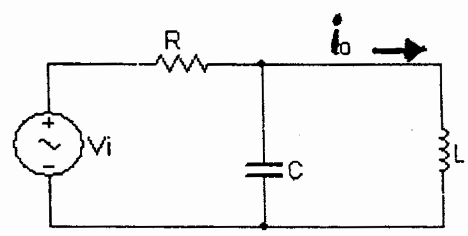

Figure 1(c)

The variables involved are as follows: r(t)= desired platform position; p(t)=actual platform position; $v_1(t)$= amplifier input voltage; $v_2(t)$=amplifier output voltage; and $\theta(t)$= motor shaft position

The initial conditions are all zero.

(i) Sketch a signal-flow diagram of the system, identifying the components parts and their transmittances. (6 marks)

(ii) Determine the system transfer function $\frac{P(s)}{R(s)}$ (5 marks)

(e) A system is described by the two differential equations

$$\frac{dy}{dt} + y - 2u + aw = 0 \quad \text{and} \quad \frac{dw}{dt} - 0.5y + 4u = 0$$

where w and y are functions of time, and u is an input.

(i) Select a set of state variables (2 marks)

(ii) write the matrix differential equation specifying the elements of the matrices. (5 marks)

(iii) solve for the parameter **a** if the characteristic roots of the system are

$$s = 0.5 \pm j3$$

(5 marks)

Question 2

Two cars with negligible friction are connected as shown in Figure 2. An input force is $u(t)$. The output [illegible] is the position of the cart 2, that is $y(t) = q(t)$. Determine a state space representation of the system.

(20 marks)

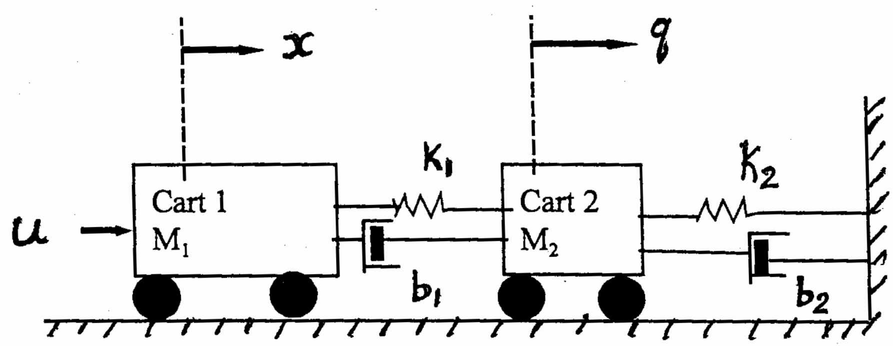

Figure 2

Question 3

For a linear system represented by the block diagram in Figure 3 the input is step of amplitude 2.4, find the following

(a) the output steady state value and the steady state error, (9 marks)

(b) the frequency of the damped oscillations, (4 marks)

(c) the percentage peak overshoot, and (4 marks)

(d) the settling time within 2% of the steady state value. (3 marks)

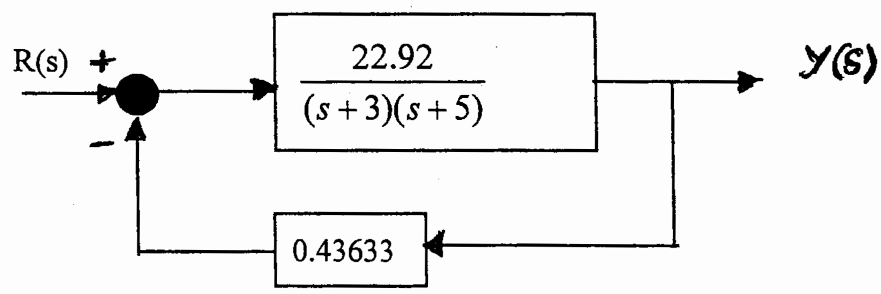

Figure 3

Question 4

Determine a state-space representation for the system shown in Figure 4. The motor inductance is negligible, the motor constant is $K_m = 10$, the back electromagnetic force constant is $K_b = 0.0706$. The motor and valve inertia is $J = 0.006$, and the capacitance of the tank is $C = 50\text{m}^2$. Note that the motor is controlled by the armature current i_a.

Let $x_1 = h$, $x_2 = \theta$ and $x_3 = \dfrac{d\theta}{dt}$ Assume that $q_1 = 80\theta$ and $Cdh = (q_i - q_o)dt$

where θ is the shaft angle. The output flow is $q_o = 50h(t)$ (20 marks)

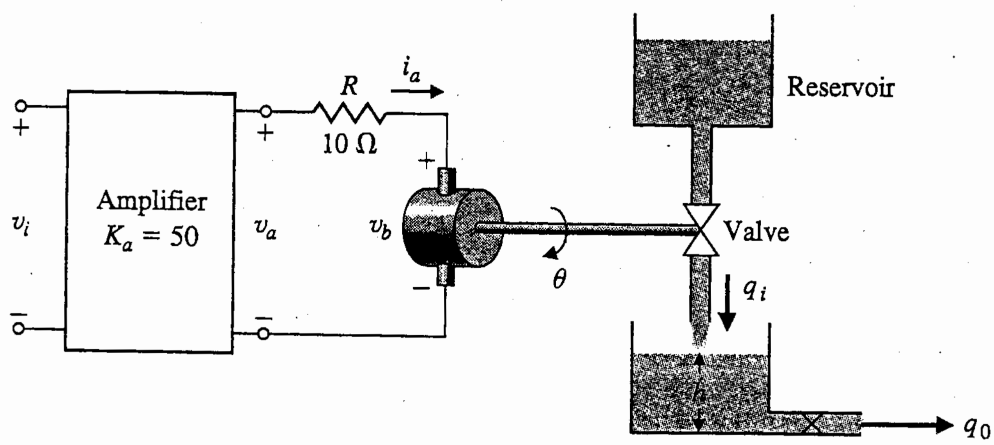

Figure 4

Question 5

A two-transistor series voltage feedback amplifier is shown in Figure 5A. The AC equivalent circuit neglects the bias resistors and the shunt capacitors. A block diagram representing the AC equivalent circuit is shown in Figure 5B. With the circuit shown in Figure 5B do the following:

(a) Using Mason signal-flow gain formula determine the voltage gain, $\frac{v_o}{v_{in}}$ (12 marks)

(b) Determine the input impedance, $\frac{v_{in}}{i_{b1}}$ (8 marks)

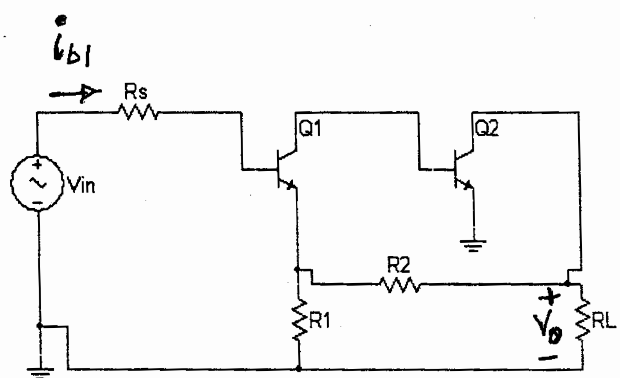

Figure 5A

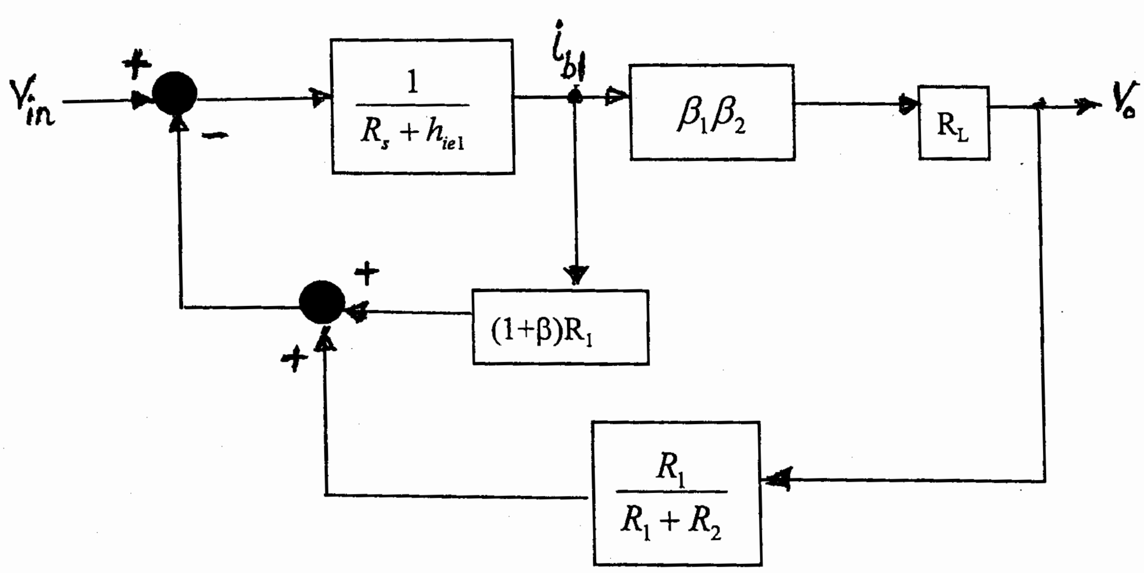

Figure 5B

Partial Table of z- and s-Transforms

	$f(t)$	$F(s)$	$F(z)$	$f(kt)$
	$u(t)$	$\frac{1}{s}$	$\frac{z}{z-1}$	$u(kT)$
	t	$\frac{1}{s^2}$	$\frac{Tz}{(z-1)^2}$	kT
	t^n	$\frac{n!}{s^{n+1}}$	$\lim_{a\to 0}(-1)^n \frac{d^n}{da^n}\left[\frac{z}{z-e^{-aT}}\right]$	$(kT)^n$
	e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$	e^{-akT}
5.	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$	$(-1)^n \frac{d^n}{da^n}\left[\frac{z}{z-e^{-aT}}\right]$	$(kT)^n e^{-akT}$
6.	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$	$\frac{z \sin \omega T}{z^2 - 2z\cos \omega T + 1}$	$\sin \omega kT$
7.	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$	$\frac{z(z-\cos \omega T)}{z^2 - 2z\cos \omega T + 1}$	$\cos \omega kT$
8.	$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$	$\frac{ze^{-aT}\sin \omega T}{z^2 - 2ze^{-aT}\cos \omega T + e^{-2aT}}$	$e^{-akT}\sin \omega kT$
9.	$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$	$\frac{z^2 - ze^{-aT}\cos \omega T}{z^2 - 2ze^{-aT}\cos \omega T + e^{-2aT}}$	$e^{-akT}\cos \omega kT$
10.			$\frac{z}{z+a}$	$a^k \cos k\pi$

z-Transform Theorems

	Theorem	Name
1.	$z\{af(t)\} = aF(z)$	Linearity theorem
2.	$z\{f_1(t) + f_2(t)\} = F_1(z) + F_2(z)$	Linearity theorem
3.	$z\{e^{-at}f(t)\} = F(e^{aT}z)$	Complex differentiation
4.	$z\{f(t-nT)\} = z^{-n}F(z)$	Real translation
5.	$z\{tf(t)\} = -Tz\frac{dF(z)}{dz}$	Complex differentiation
6.	$f(0) = \lim_{z\to\infty} F(z)$	Initial value theorem
7.	$f(\infty) = \lim_{z\to 1}(1-z^{-1})F(z)$	Final value theorem