UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRONIC ENGINEERING

MAIN EXAMINATION 2007

TITLE OF PAPER	**:**	**ORDINARY DIFFERENTIAL EQUATIONS, PROBABILITY AND STATISTICS**
COURSE NUMBER	**:**	**E371**
TIME ALLOWED	**:**	**THREE HOURS**
INSTRUCTIONS	**:**	**ANSWER ANY FOUR OUT OF FIVE QUESTIONS. EACH QUESTION CARRIES 25 MARKS. MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.**

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E371 ORDINARY DIFFERENTIAL EQUATIONS, PROBABILITY AND STATISTICS

Question one

Given the following inhomogeneous second order ordinary differential equation as :

$$4 \frac{d^2 f(t)}{dt^2} + 2 \frac{d f(t)}{dt} + f(t) = 2t - 3t^2$$

(a) set the particular solution of $f(t)$ as $k_1 + k_2 t + k_3 t^2$, find the values of k_1 , k_2 *and* k_3 and thus the particular solution of $f(t)$, namely $f_p(t)$,

(8 marks)

(b) find the general solution of the homogeneous part of the given equation , i.e.,

$4 \frac{d^2 f(t)}{dt^2} + 2 \frac{d f(t)}{dt} + f(t) = 0$, and name it as $f_h(t)$, (5 marks)

(c) (i) write down the general solution of the above inhomogeneous equation in terms of $f_p(t)$ and $f_h(t)$, and name it as $f_g(t)$, (2 marks)

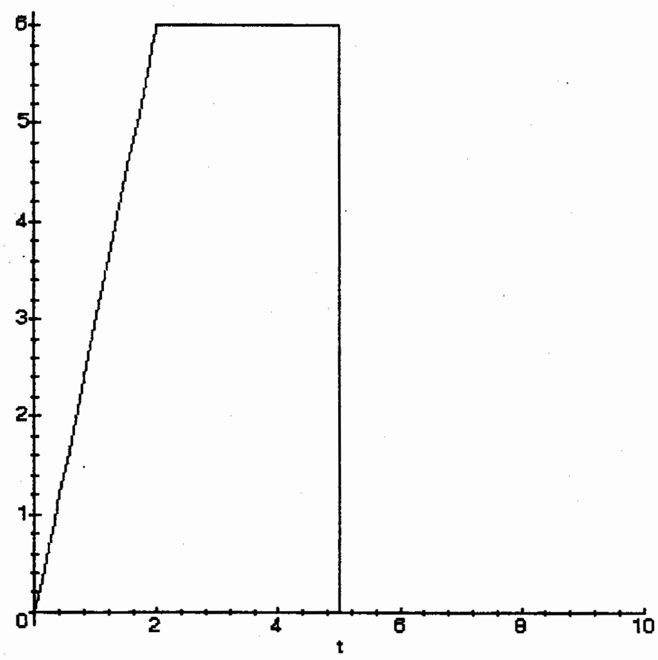
(ii) if the initial conditions are given as $f(0) = 9$ and $f'(0) = 2$, determine the values of the arbitrary constants in $f_g(t)$ and thus the specific solution of $f(t)$, name it as $f_s(t)$. Plot both $f_s(t)$ and $f_p(t)$ for $t = 0$ *to* 8 , and show them in a single display. Compare their behaviour at large t and make a brief remark. (10 marks)

Question two

Given the following inhomogeneous second order ordinary differential equation as :

$$\frac{d^2 f(t)}{dt^2} + 3\frac{d f(t)}{dt} + 9 f(t) = g(t)$$

(a) (i) if $g(t)$ is a pulse function and is given as follows :



(i.e., $g(t) = 0$ *for* $t \le 0$ *and* $t \ge 5$ and the peak value of $g(t)$ is 6 happened at $t = 2$ *to* 5)

write down the above pulse function of t in terms of Heaviside functions and plot it for $t = 0$ *to* 10 to reproduce the above diagram. (5 marks)

(ii) find the Laplace transform of $g(t)$ given in (a) (i) and named it as $G(s)$.

(2 marks)

(b) (i) find, $F(s)$, the Laplace transform of $f(t)$ if $f(0) = 7$ and $f'(0) = 3$.

Show that $F(s)$ can be rewritten as $F(s) = K(s) + H(s)\,G(s)$

where $G(s)$ is obtained in (a)(ii), $K(s) = \dfrac{7s + 24}{s^2 + 3s + 9}$ and

$H(s) = \dfrac{1}{s^2 + 3s + 9}$ (7 marks)

(ii) find the inverse Laplace transform of $K(s)$ and $H(s)$, and name them as $k(t)$ and $h(t)$ respectively, (3 marks)

(iii) find the convolution of $h(t)$ and $g(t)$, and name it as $hg(t)$,

(5 marks)

(iv) write down the specific solution of $f(t)$ in terms of $k(t)$ and $hg(t)$ and plot it for $t = 0$ *to* 10. (3 marks)

Question three

(a) Given the system of linear equations in matrix form as $A\,X = b$ where

$$A = \begin{pmatrix} -1 & 2 & -3 \\ 1 & -4 & 3 \\ 2 & 5 & -7 \end{pmatrix}, \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad b = \begin{pmatrix} -21 \\ 15 \\ -9 \end{pmatrix},$$

(i) *augment* A *and* b, then apply the Gauss elimination method using commands of *addrow and backsub* to find the solution of X.

(5 marks)

(ii) use the Cramer's rule to find the solution X. Compare the answer obtained here with that obtained in (a)(i). (5 marks)

(b) Given the following system of differential equations for coupled oscillators as :

$$\begin{cases} \dfrac{d^2 x_1(t)}{dt^2} = -16\,x_1(t) + 5\,x_2(t) \\ \dfrac{d^2 x_2(t)}{dt^2} = 2\,x_1(t) - 7\,x_2(t) \end{cases}$$

(i) set $x_1(t) = X_1\,e^{i\omega t}$ and $x_2(t) = X_2\,e^{i\omega t}$, deduce the following matrix equation $-\omega^2 \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = A \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$ where $A = \begin{pmatrix} -16 & 5 \\ 2 & -7 \end{pmatrix}$

(4 marks)

(ii) find the eigenvalues and eigenvectors of A and thus evaluate the eigenfrequencies ω, (6 marks)

(iii) construct a matrix B by augmenting the eigenvectors obtained in (b)(ii) and show that the similarity transformation of A by B yields a diagonalized matrix. (5 marks)

Question four

Given the following differential equation :

$$(1-x^2)\frac{d^2y(x)}{dx^2} - 2x\frac{dy(x)}{dx} + 6y(x) = 0$$

(a) set $y(x) = \sum_{n=0}^{\infty} a_n x^{n+s}$ *and* $a_0 \neq 0$, use the power series method to find the indicial equations and solve for the values of s and possibly the value of a_1 ,

(6 marks)

(b) find the recurrence relation, (3 marks)

(c) for each values of s found in (a) , set $a_0 = 1$ and find the values of a_1 , a_2 , a_3, ... , a_{10} by using the recurrence relation in (b). Then write down two particular solutions expressed in power series and truncated to the a_{10} term.

(8 marks)

(d) write down the general solutions in terms of those two truncated particular solutions. If initially $y(0) = 5$ and $\left.\frac{dy(x)}{dx}\right|_{x=0} = 3$, determine the values of the arbitrary constants in the general solution and thus obtain the specific solution. Plot the specific solution for $x = 0$ *to* 1 . (8 marks)

Question five

(a) (i) Use the random number generator in MAPLE to generate an ensemble S of 20 data values ranging from 20 *to* 79 , (4 marks)

(ii) find the values of mean , variance and standard deviation of S (5 marks)

(iii) use the interval of 10 , starting from 19.5 and ending at 79.5 , i.e., $(19.5, 29.5), (29.5, 39.5), \cdots\cdots$, to plot a histogram of S (8 marks)

(c) For a normal distribution $f(x)$ with the mean value of 12 and the standard deviation of 7 ,

(i) plot $f(x)$ for $x = 0$ *to* 20 , (3 marks)

(ii) find its corresponding cumulative distribution function $g(x)$ and use it to calculate the values of the probabilities of $P(x > 5)$ and $P(4 < x < 10)$.

(5 marks)