UNIVERSITY OF SWAZILAND

MAIN EXAMINATION, MAY 2007

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRONIC ENGINEERING

TITLE OF PAPER:	DIGITAL SIGNAL PROCESSING
COURSE CODE:	E420
TIME ALLOWED:	THREE HOURS

INSTRUCTIONS:

1. **Answer any FOUR (4) of the following five questions.**
2. **Each question carries 25 marks.**
3. **Tables of selected window functions and selected Z-transform pairs are attached.**

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THIS PAPER CONTAINS EIGHT (8) PAGES INCLUDING THIS PAGE

QUESTION ONE ***(25 marks)***

(a) An audio signal whose spectrum extends up to $f_c = 10$ kHz is filtered with a filter whose response is

$$H(f) = \frac{1}{\sqrt{1+\left(\frac{f}{f_c}\right)^{10}}}$$

It is then digitized using a 10-bit ADC with uniform quantization.
What minimum sampling frequency should be used if the maximum aliasing error is to be less than the r.m.s quantization noise level? *(8 marks)*

(b) The Z-transform of a discrete sequence $x(n)$ is $X(z) = \frac{z^2}{(z-0.4)(z+0.6)}$

Obtain the first 4 terms of the sequence using two different methods and show that the two methods chosen give the same sequence. *(17 marks)*

QUESTION TWO **(25 *marks)***

(a) The frequency response of a discrete system is given by $H(e^{j\omega T}) = e^{-j\omega T/4}$.

(i) Find an expression for the phase angle of the system. *(1 mark)*

(ii) Sketch the phase angle for $-f_s \leq f \leq f_s$. *(2 marks)*

(iii) If h(n) is the impulse response of the system, determine:

a. h(0) *(6 marks)*

b. $\sum_{n=-\infty}^{\infty} h(n)$ *(2 marks)*

c. $\sum_{n=-\infty}^{\infty} [h(n)]^2$ *(2 marks)*

Hint: You should be able to obtain these without obtaining an explicit expression for h(n).

(b) A moving average filter has a difference equation

$$y(n) = \frac{1}{3}\left[x(n) + x(n-1) + x(n-2)\right]$$

(i) Find and sketch its amplitude and phase response. *(7 marks)*

(ii) Evaluate its group delay. *(1 mark)*

(c) An analogue signal sampled at 8 kHz is to be filtered using a digital band pass filter with a pass band between 3020 Hz and 3140 Hz. Specify the pass band of the digital filter.

(4 marks)

QUESTION THREE **(25 *marks*)**

A structure of a non-recursive filter is shown in Fig. Q.3

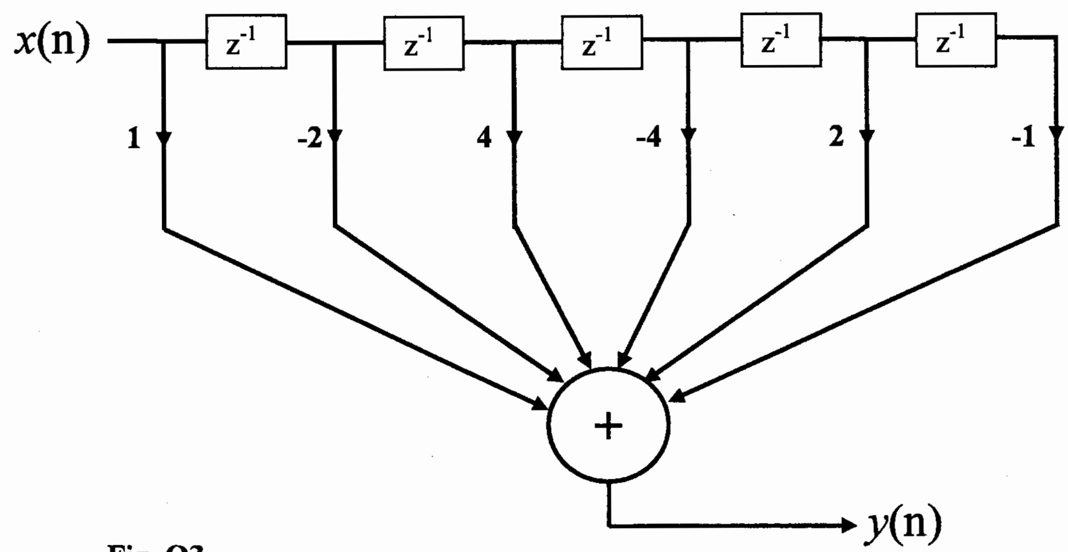

Fig. Q3

(a) Find and sketch its unit impulse response. *(3 marks)*

(b) Find and sketch its unit step response. *(4 marks)*

(c) Explain why the unit step response settles to the value found in (b). *(2 marks)*

(d) What type of filtering action does the filter produce and why? *(2 marks)*

(e) Find an expression for its amplitude and phase response. *(8 marks)*

(f) A 5-kHz sinusoidal signal of unit amplitude is sampled at 20 kHz and passed through the filter. Find the amplitude and phase of the output signal. *(6 marks)*

QUESTION FOUR **(25 *marks*)**

(a) Given two sequences:

[1, -1, -1, 1] and [1, 0, 0, 1]

whose DFTs are

[0, 2+j2, 0, 2-j2] and [2, 1+j, 0, 1-j] respectively.

(i) Determine the circular cross-correlation by a direct method. *(3 marks)*

(ii) Determine the cross correlation using the Fast Correlation method . *(5 marks)*

(b) A digital filter has poles at $0.5 \pm j0.5$ and $0.5 \pm j0.2$ and zeros at $0.3 \pm j0.8$. By using a partial fraction method obtain a realization structure for the filter as a cascade of second or first order structures i.e. $\frac{N_1(z)}{D_1(z)}\frac{N_2(z)}{D_2(z)}$ where $N_i(z)$ and $D_i(z)$ are first or second order polynomials. *(6 marks)*

(c) A filter response is represented by $H(z) = \frac{z^2 - 0.7z + 0.49}{z^2 - 0.9z + 0.81}$. What type of response does this represent, lowpass, highpass, bandpass or bandstop? Justify your answer. *(6 marks)*

(d) Obtain the difference equation relating y(n) and x(n) for the filter structure in Fig. Q4. Is the filter FIR or IIR? *(5 marks)*

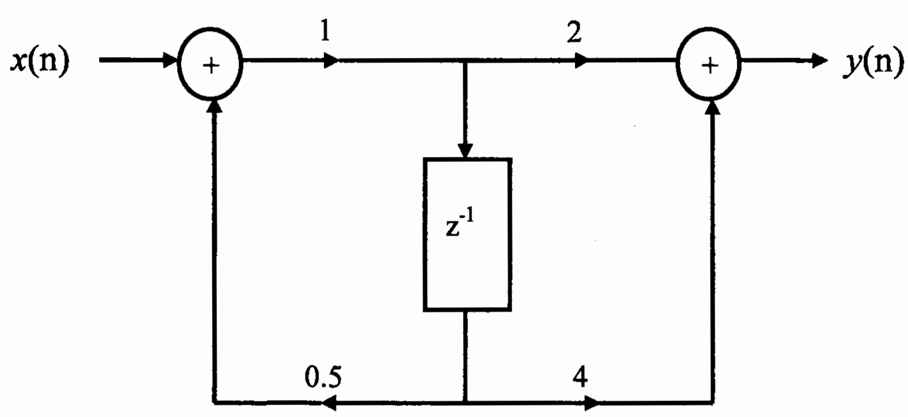

Fig. Q.4

QUESTION FIVE **(25 *marks*)**

(a) Which considerations would you undertake when choosing whether to use an FIR or IIR filter in a particular application. Explain why you think these considerations are necessary. *(6 marks)*

(b) A lowpass audio FIR filter with the following specification is to be designed using the window method:

Pass band 0 Hz to 10 kHz

Minimum attenuation ay 12 kHz is 43 dB

Sampling rate 24 kHz

(i) Give an expression for the impulse response of an ideal lowpass filter in terms of its cut off frequency. *(1 mark)*

(ii) Select a suitable window from the attached table which meets the required specification. *(2 marks)*

(iii) Obtain the minimum order of the filter required. *(2 marks)*

(iv) If the filter is linear phase, find an expression for its coefficients and determine the first three coefficients of the filter. *(6 marks)*

(c) Design a low pass IIR filter with a cut off frequency of 40 Hz with a sampling rate of 240 Hz. Your design should be based on a second order Butterworth analogue filter which has a normalized transfer function

$$H(s) = \frac{1}{(s^2 + \sqrt{2}\,s + 1)}.$$ *(8 marks)*

TABLE OF Z-TRANSFORMS OF SOME COMMON SEQUENCES

Discrete-time sequence $x(n),\ n \geq 0$	**Z-transform** $H(z)$
$k\delta(n)$	k
k	$\frac{kz}{z-1}$
$ke^{-\alpha n}$	$\frac{kz}{z-e^{-\alpha}}$
$k\alpha^n$	$\frac{kz}{z-\alpha}$
kn	$\frac{kz}{(z-1)^2}$
kn^2	$\frac{kz(z+1)}{(z-1)^3}$
$kn\alpha^n$	$\frac{k\alpha z}{(z-\alpha)^2}$

SUMMARY OF IMPORTANT FEATURES OF SELECTED WINDOW FUNCTIONS

Name of Widow	Normalized Transition Width	Passband Ripple (dB)	Main lobe relative to Sidelobe (dB)	Max. Stopband attenuation (dB)	6 dB normalized bandwidth (bins)	Window Function $\omega(n)$, $\lvert n\rvert \le (N-1)/2$
Rectangular	0.9/N	0.7416	13	21	1.21	1
Hanning	3.1/N	0.0546	31	44	2.00	$0.5+0.5\cos\left(\frac{2\pi n}{N}\right)$
Hamming	3.3/N	0.0194	41	53	1.81	$0.54+0.46\cos\left(\frac{2\pi n}{N}\right)$
Blackman	5.5/N	0.0017	57	74	2.35	$0.42+0.5\cos\left(\frac{2\pi n}{N-1}\right)+0.08\cos\left(\frac{4\pi n}{N-1}\right)$
Kaiser	2.93/N (β=4.54)	0.0274		50		$\frac{I_o\left(\beta\left\{1-\left[\frac{2n}{N-1}\right]^2\right\}^{\frac{1}{2}}\right)}{I_o(\beta)}$
	4.32/N (β=6.76	0.00275		70		
	5.71/N (β=8.96)	0.000275		90		

$$\text{Bin width} = \frac{f_s}{N}\ \text{Hz}$$