

**UNIVERSITY OF SWAZILAND  
FACULTY OF SCIENCE  
DEPARTMENT OF ELECTRONIC ENGINEERING**

**SUPPLEMENTARY EXAMINATION JULY 2007**

**TITLE OF PAPER: CONTROL SYSTEMS**

**COURSE CODE: E430**

**TIME ALLOWED: THREE HOURS**

**INSTRUCTIONS:**

1. Answer all four questions.
2. Each Question carries 25 marks.
3. Marks for different sections are shown in the right-hand margin

This paper has 6 pages including this page.

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BY THE INVIGILATOR.**

Partial Table of z- and s-Transforms

|     | $f(t)$                  | $F(s)$                              | $F(z)$  | $f(kT)$                   |
|-----|-------------------------|-------------------------------------|---|---------------------------|
| 1.  | $u(t) = 1$              | $\frac{1}{s}$                       | $\frac{z}{z-1}$   | $u(kT)$                   |
| 2.  | $t$                     | $\frac{1}{s^2}$                     | $\frac{Tz}{(z-1)^2}$  | $kT$                      |
| 3.  | $t^n$                   | $\frac{n!}{s^{n+1}}$                | $\lim_{a \rightarrow 0} (-1)^n \frac{d^n}{da^n} \left[ \frac{z}{z - e^{-aT}} \right]$ | $(kT)^n$                  |
| 4.  | $e^{-at}$               | $\frac{1}{s+a}$                     | $\frac{z}{z - e^{-aT}}$   | $e^{-akT}$                |
| 5.  | $t^n e^{-at}$           | $\frac{n!}{(s+a)^{n+1}}$            | $(-1)^n \frac{d^n}{da^n} \left[ \frac{z}{z - e^{-aT}} \right]$                        | $(kT)^n e^{-akT}$         |
| 6.  | $\sin \omega t$         | $\frac{\omega}{s^2 + \omega^2}$     | $\frac{z \sin \omega T}{z^2 - 2z \cos \omega T + 1}$                                  | $\sin \omega kT$          |
| 7.  | $\cos \omega t$         | $\frac{s}{s^2 + \omega^2}$          | $\frac{z(z - \cos \omega T)}{z^2 - 2z \cos \omega T + 1}$                             | $\cos \omega kT$          |
| 8.  | $e^{-at} \sin \omega t$ | $\frac{\omega}{(s+a)^2 + \omega^2}$ | $\frac{ze^{-aT} \sin \omega T}{z^2 - 2ze^{-aT} \cos \omega T + e^{-2aT}}$             | $e^{-akT} \sin \omega kT$ |
| 9.  | $e^{-at} \cos \omega t$ | $\frac{s+a}{(s+a)^2 + \omega^2}$    | $\frac{z^2 - ze^{-aT} \cos \omega T}{z^2 - 2ze^{-aT} \cos \omega T + e^{-2aT}}$       | $e^{-akT} \cos \omega kT$ |
| 10. |                         |                                     | $\frac{z}{z+a}$   | $a^k \cos kT$             |

z-Transform Theorems

| Theorem  | Name                    |
|--|-------------------------|
| 1. $z\{af(t)\} = aF(z)$                                  | Linearity theorem       |
| 2. $z\{f_1(t) + f_2(t)\} = F_1(z) + F_2(z)$              | Linearity theorem       |
| 3. $z\{e^{-at} f(t)\} = F(e^{aT} z)$                     | Complex differentiation |
| 4. $z\{f(t - nT)\} = z^{-n} F(z)$                        | Real translation        |
| 5. $z\{t f(t)\} = -Tz \frac{dF(z)}{dz}$                  | Complex differentiation |
| 6. $f(0) = \lim_{z \rightarrow \infty} F(z)$             | Initial value theorem   |
| 7. $f(\infty) = \lim_{z \rightarrow 1} (1 - z^{-1})F(z)$ | Final value theorem     |

Note:  $kT$  may be substituted for  $t$  in the table.

**Question 1**

A) The transfer function of a system is  $\frac{Y(s)}{R(s)} = \frac{10(s+2)}{s^2+8s+15}$  determine response  $y(t)$  when the input  $r(t)$  is unit step. [8 marks]

B) Draw a signal flow graph for the system shown in Figure 1 and then use Mason's gain rule to find the transfer function for this system. [17 marks]

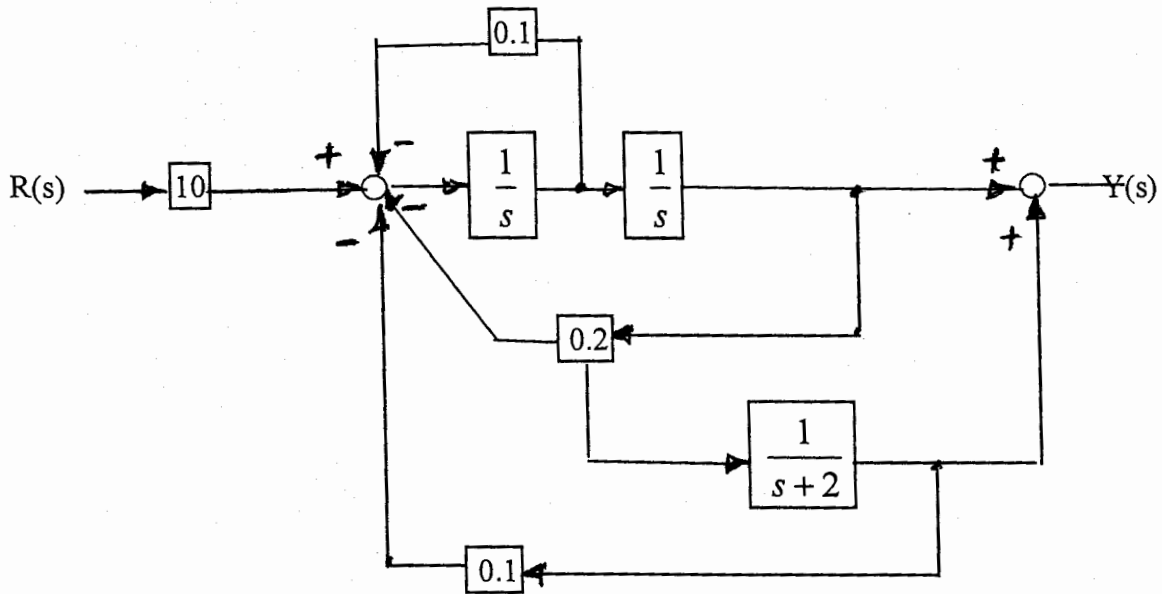


Figure 1

**Question 2**

An RLC network is shown in Figure 2. Define the state variables as  $x_1 = i_L$ ,  $x_2 = v_C$ , input variables as  $v_1 = u_1$ ,  $v_2 = u_2$ , and output variable as  $v_R = y$ . Obtain the state differential equations [25 marks]

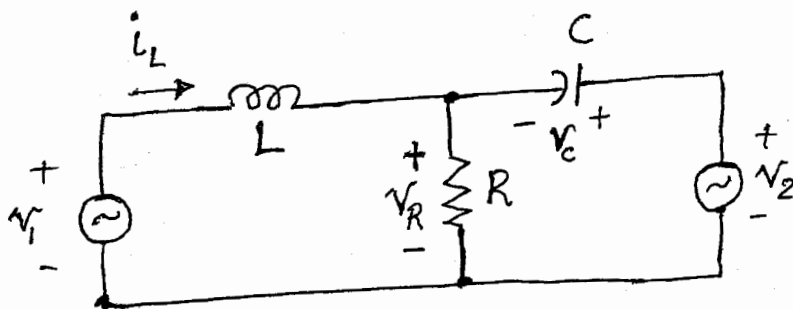


Figure 2

**Question 3**

For a second order system shown in Figure 3 A, with the input being a unit step,

- A) determine the steady state error [6 marks]
- B) determine the damping ratio and the maximum percentage overshoot [6 marks]
- C) determine the value of gain  $K$ , as shown in Figure 3 B, that will make the system to have a 1% maximum overshoot, and what will the steady state error be [13 marks]

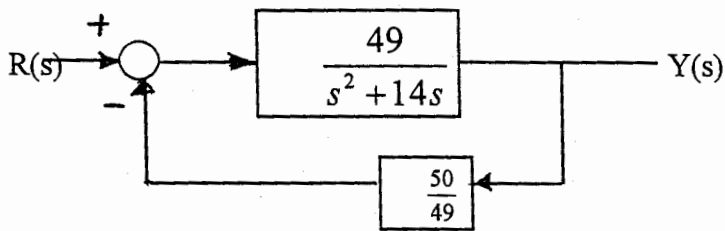


Figure 3 A

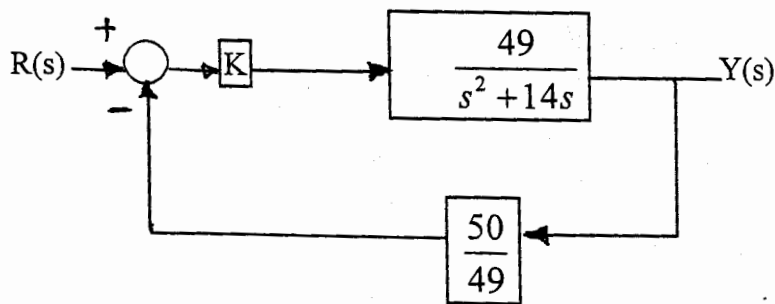


Figure 3B

**Question 4**

Draw the Bode plot ( Magnitude and Phase) of

$$G(s) = \frac{12 \cdot 10^5 (s + 100)^2}{(s + 10)^2 (s + 1200)}$$