UNIVERSITY OF SWAZILAND
FACULTY OF SCIENCE
DEPARTMENT OF ELECTRONIC ENGINEERING

MAIN EXAMINATION 2006/2007

TITLE OF PAPER : SIGNALS II

COURSE NUMBER : E462

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS : ANSWER ANY FOUR OUT OF THE FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN

USEFUL INFORMATION IS ATTACHED AT THE END OF THE EXAMINATION PAPER

DO NOT OPEN THE PAPER UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR

QUESTION ONE

(a) Determine if each of the following is a probability density function or not

(i) $3\prod(t-1)$ (3 marks)

(ii) $2t\prod(t-0.5)$ (3 marks)

(b) If the probability of a single digit error is 0.01, calculate the probability of having more than 2 errors occurring in a 10-digit codeword. Assume that all events are statistically independent.

(6 marks)

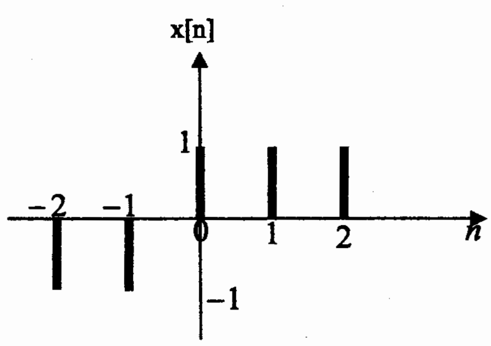
(c) Calculate the inverse Fourier Transform for the following signal (4 marks)

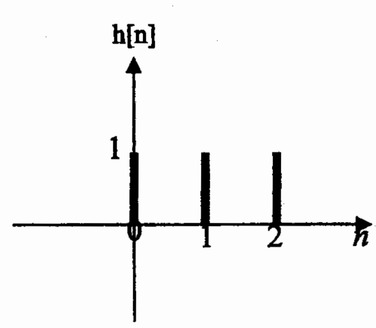
$$X(\omega)=\begin{cases} k & -B\leq\omega\leq B \\ 0 & elsewhere \end{cases}$$

(d) Given that $x(t)=\begin{cases} e^{-kt} & t>0 \\ 0 & t<0 \end{cases}$, determine the Fourier Transform of $x(t-\beta)$

(3 marks)

(e) Let the signals x[n] and h[n] represent the signals shown below





Both signals are zero outside the indicated range of n. Sketch $x[n]*h[n]$

(6 marks)

QUESTION TWO

(a) Determine the energy spectral densities of the following functions

(i) $\prod(t-2)$ (4 marks)

(ii) $\prod(4t)$ (4 marks)

(b) Evaluate and plot the magnitude spectrum of the 8-point DFT for the following signal. The sampling frequency is 8 KHz

(12 marks)

$x(t) = \sin(2\pi 1300t)$ volts

(c) A random voltage has a *pdf* given by

$$pdf(\mathrm{V}) = 0.45\partial(\mathrm{V}-3) + x\mathbf{u}(\mathrm{V}+3)\,e^{-3(V+3)}$$

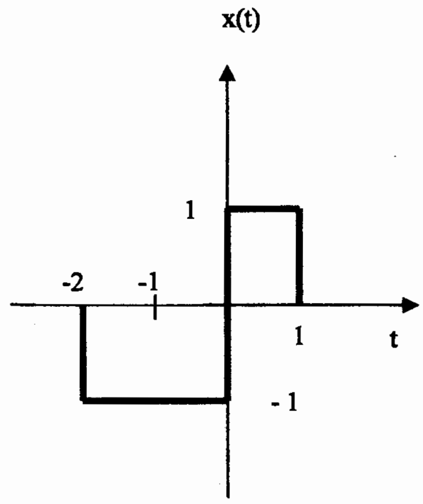
where **u**( ) is a unit step function

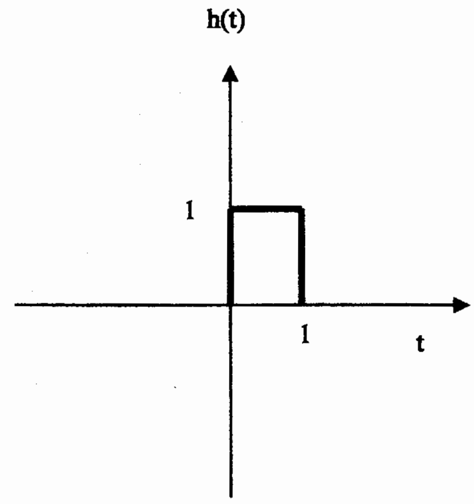
(i) find the probability that V = 3 (1 marks)

(ii) find the value of x (4 marks)

QUESTION THREE

(a) Given the following two signals x(t) and h(t), sketch their convolution result (10 marks)





(b) A random variable has a *pdf* given by

$pdf(V) = \mathbf{u}(V)3e^{-3V}$, where $\mathbf{u}(\,)$ is a unit step function

(i) Find the DC value (3 marks)
(ii) Find the power dissipated in a 1Ω load (5 marks)

(c) Given the Fourier Transform pair $x(t) \leftrightarrow X(\omega)$, Show that

$$\frac{dx}{dt} \leftrightarrow j\omega X(\omega)$$

(3 marks)

(d) An N-sample signal $x[n]$ has the DFT $X[k]$. Write down the expression for the DFT of the following signals

(i) $2x[n] + x[n+1]$ (2 marks)
(ii) $x[n]x[n-1]$ (2 marks)

QUESTION FOUR

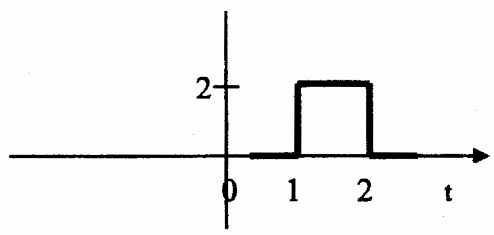
(a) From the definition of the Fourier Transform, show that

$$x(t - t_0) \leftrightarrow e^{-j\omega t_0} X(\omega)$$ (5 marks)

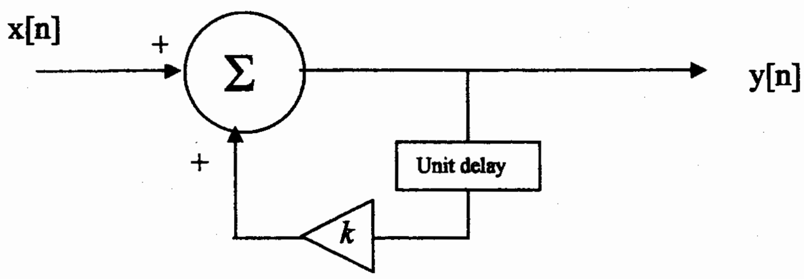
(b) Sketch the autocorrelation function of the following rectangular pulse

(10 marks)

$$x(t) = 2\prod(t - 1.5)$$



(c) A processor is shown in the figure below

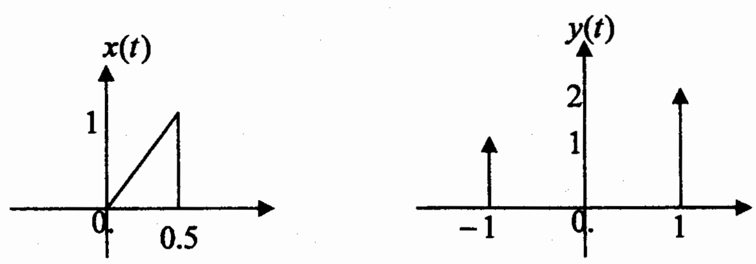


Find its digital transfer function and the first four output values for a weighted unit-impulse input $2\partial[n]$ for $k = 0.5$

(7 marks)

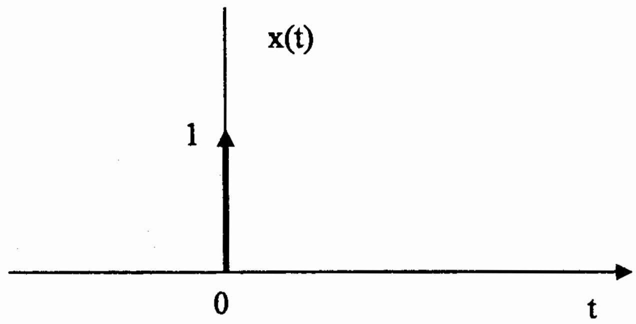
(d) Sketch the convolution of the two functions shown below

(3 marks)



QUESTION FIVE

(a) Find and sketch the Fourier Transform of the following signal (3 marks)



(b) Find x[n], $0 \leq n \leq 3$, using long division for X(z) (5 marks)

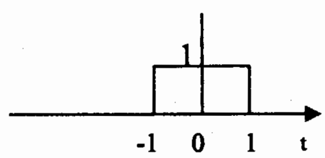
$$\frac{1+2z^{-1}}{1+2z^{-1}+4z^{-2}}$$

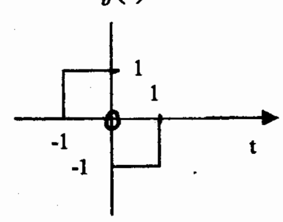
(c) A digital processor is described by a unit impulse response, h[n] = [3, 4, 2, 3]. If the input sequence is x[n] = [2, 1, 3, 0, 5], determine the output, y[n], by using circular convolution

(5 marks)

(d) Find the Fourier transform of the following signal (10 marks)

$$x(t) = 0.5[x_a(t) - x_b(t)]$$





(e) How does a Butterworth filter approximate an ideal, 'rectangular' response characteristic

(2 marks)

Table 1.. Properties of the Fourier Transform

Property	Signal	Fourier transform
	$x(t)$	$X(\omega)$
	$x_1(t)$	$X_1(\omega)$
	$x_2(t)$	$X_2(\omega)$
Linearity	$a_1x_1(t)+a_2x_2(t)$	$a_1X_1(\omega)+a_2X_2(\omega)$
Time shifting	$x(t-t_0)$	$e^{-j\omega t_0}X(\omega)$
Frequency shifting	$e^{j\omega_0 t}x(t)$	$X(\omega-\omega_0)$
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$
Time reversal	$x(-t)$	$X(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Time differentiation	$\frac{dx(t)}{dt}$	$j\omega X(\omega)$
Frequency differentiation	$(-jt)x(t)$	$\frac{dX(\omega)}{d\omega}$
Integration	$\int_{-\infty}^{t} x(\tau)\,d\tau$	$\pi X(0)\delta(\omega)+\frac{1}{j\omega}X(\omega)$
Convolution	$x_1(t)*x_2(t)$	$X_1(\omega)X_2(\omega)$
Multiplication	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(\omega)*X_2(\omega)$
Real signal	$x(t)=x_e(t)+x_o(t)$	$X(\omega)=A(\omega)+jB(\omega)$
		$X(-\omega)=X^*(\omega)$
Even component	$x_e(t)$	$\mathrm{Re}\{X(\omega)\}=A(\omega)$
Odd component	$x_o(t)$	$j\,\mathrm{Im}\{X(\omega)\}=jB(\omega)$
Parseval's relations		

$$\int_{-\infty}^{\infty} x_1(\lambda)X_2(\lambda)\,d\lambda=\int_{-\infty}^{\infty} X_1(\lambda)x_2(\lambda)\,d\lambda$$

$$\int_{-\infty}^{\infty} x_1(t)x_2(t)\,dt=\frac{1}{2\pi}\int_{-\infty}^{\infty} X_1(\omega)X_2(-\omega)\,d\omega$$

$$\int_{-\infty}^{\infty} |x(t)|^2\,dt=\frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\,d\omega$$

Table 2 Some Common z-Transform Pairs

$x[n]$	$X(z)$	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1-z^{-1}},\frac{z}{z-1}$	$\lvert z\rvert>1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}},\frac{z}{z-1}$	$\lvert z\rvert<1$
$\delta[n-m]$	z^{-m}	All z except 0 if $(m>0)$ or ∞ if $(m<0)$
$a^n u[n]$	$\frac{1}{1-az^{-1}},\frac{z}{z-a}$	$\lvert z\rvert>\lvert a\rvert$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}},\frac{z}{z-a}$	$\lvert z\rvert<\lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2},\frac{az}{(z-a)^2}$	$\lvert z\rvert>\lvert a\rvert$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2},\frac{az}{(z-a)^2}$	$\lvert z\rvert<\lvert a\rvert$
$(n+1)a^n u[n]$	$\frac{1}{(1-az^{-1})^2},\left[\frac{z}{z-a}\right]^2$	$\lvert z\rvert>\lvert a\rvert$
$(\cos\Omega_0 n)u[n]$	$\frac{z^2-(\cos\Omega_0)z}{z^2-(2\cos\Omega_0)z+1}$	$\lvert z\rvert>1$
$(\sin\Omega_0 n)u[n]$	$\frac{(\sin\Omega_0)z}{z^2-(2\cos\Omega_0)z+1}$	$\lvert z\rvert>1$
$(r^n\cos\Omega_0 n)u[n]$	$\frac{z^2-(r\cos\Omega_0)z}{z^2-(2r\cos\Omega_0)z+r^2}$	$\lvert z\rvert>r$
$(r^n\sin\Omega_0 n)u[n]$	$\frac{(r\sin\Omega_0)z}{z^2-(2r\cos\Omega_0)z+r^2}$	$\lvert z\rvert>r$
$\begin{cases}a^n & 0\le n\le N-1\\ 0 & \text{otherwise}\end{cases}$	$\frac{1-a^Nz^{-N}}{1-az^{-1}}$	$\lvert z\rvert>0$

TABLE 3 FOURIER TRANSFORMS OF ELEMENTARY FUNCTIONS

Continuous Time Function $x(t)$	Fourier Transform $X(j\omega)$	Remark
1	$2\pi\delta(\omega)$	Constant, noncausal.
$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	Unit-step function, causal.
$\delta(t)$	1	Delta distribution, noncausal.
$\delta(t-t_0)$	$\exp(-j\omega t_0)$	Delayed delta distribution, noncausal.
$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\frac{2\pi}{T}\sum_{n=-\infty}^{\infty}\delta\left(\omega-\frac{2n\pi}{T}\right)$	Impulse train.
$\mathrm{rect}(t/\tau)$	$\frac{2\sin(\omega\tau/2)}{\omega}=\tau\,\mathrm{sinc}(\omega\tau/2)$	Rectangular pulse, noncausal.
$\frac{\sin(\omega_0 t)}{\pi t}=\frac{\omega_0}{\pi}\,\mathrm{sinc}\left(\omega_0 t\right)$	$\mathrm{rect}\left(\frac{\omega}{2\omega_0}\right)$	Noncausal.
$\exp(j\omega_0 t)$	$2\pi\delta(\omega-\omega_0)$	Complex exponential, noncausal.
$\cos(\omega_0 t)$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	Noncausal.
$\sin(\omega_0 t)$	$\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$	Noncausal.
$\cos(\omega_0 t)u(t)$	$\frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\frac{j\omega}{\omega_0^2-\omega^2}$	Causal.
$\sin(\omega_0 t)u(t)$	$\frac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$	Causal.
$\exp(-at)u(t)$	$\frac{1}{a+j\omega}$	Re[a]>0, causal.
$t\exp(-at)u(t)$	$\frac{1}{(a+j\omega)^2}$	Re[a]>0, causal.
$\exp(-a\|t\|)$	$\frac{2a}{a^2+\omega^2}$	Re[a]>0, noncausal.
$\|t\|\exp(-a\|t\|)$	$\frac{2(a^2-\omega^2)}{a^2+\omega^2}$	Noncausal.

Table 4: SOME PROPERTIES OF THE DFT

1. Linearity

$$a_1x_1[n]+a_2x_2[n] \leftrightarrow a_1X_1[k]+a_2X_2[k]$$

2. Time-shifting

$$x[n-n_0] \leftrightarrow X[k]e^{-j\frac{2\pi kn_0}{N}} = X[k]W_N^{kn_0}$$

3. Modulation/Multiplication

$$x_1[n]x_2[n] \leftrightarrow \frac{1}{N}\sum_{m=0}^{N-1} X_1[m]X_2[k-m]$$

4. Frequency Shifting

$$W_N^{-kn_0}x[n] \leftrightarrow X[k-k_0]$$

5. Time reversal

$$x[-n] \leftrightarrow X[-k]$$

6. Convolution

$$\sum_{m=0}^{N-1} x_1[n]x_2[m-n] \leftrightarrow X_1[k]X_2[k]$$