UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS & ELECTRONIC ENGINEERING

MAIN EXAMINATION 2007

TITLE OF PAPER	:	COMPLEX VARIABLES
COURSE NUMBER	:	E471
TIME ALLOWED	:	THREE HOURS
INSTRUCTIONS	:	ANSWER ANY FOUR OUT OF FIVE QUESTIONS. EACH QUESTION CARRIES 25 MARKS. MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.

THIS PAPER HAS SEVEN PAGES, INCLUDING THIS PAGE.

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Question one

(a) Convert the given complex number $\dfrac{\sin(2+i)}{(5-3i)}$ into its polar form and express its phase angles in terms of degrees. (4 marks)

(b) Given the following complex function $f(z)=\dfrac{z^2-2e^z+1}{(z+5)}$ where $z=x+iy$,

(i) find its $u(x,y)$ *and* $v(x,y)$, (3 marks)

(ii) check for its analyticity , (4 marks)

(iii) plot the mapped image of $u(x,y)=2$ and $u(x,y)=4$ curves onto the $z-plane$ and show them in one display in $z-plane$ for $x=-3$ *to* $+3$ and $y=-3$ *to* $+3$. (4 marks)

(Note : $f(z)=u(x,y)+iv(x,y)$)

(c) Determine the values of a such that $u(x,y)=e^{ax}\cos(5y)$ is a harmonic and then find its conjugate harmonic $v(x,y)$ for each values of a .

(10 marks)

Question two

(a) Evaluate the value of the following complex line integral $\int_C \frac{z+3}{(z+4)(z-6)}\,dz$ if

(i) C : the shortest path from the origin to $-6-8i$, **(7 marks)**

(ii) C : the counter clockwise circular path from the origin to $-6-8i$ with the centre of the circle at $-3-4i$ and the radius of 5 .

(Hint : set $x = -3 + 5\cos(t)$, $y = -4 + 5\sin(t)$ where $t = \tan^{-1}\left(\frac{4}{3}\right)$ *to* $\pi + \tan^{-1}\left(\frac{4}{3}\right)$)

Compare the answer here with that obtained in (a)(i) and make brief comment. **(10 marks)**

(b) Given a complex function $f(z) = \frac{3}{2z+6}$ and the expansion centre as $z = 4i$, divide the space into two regions and find the convergent series expansion of $f(z)$ in those two regions. **(8 marks)**

Question three

(a) **Evaluate the value of the following contour integral**

$$\oint_C \frac{z \cosh\left(\frac{\pi z}{9}\right)}{z^4 + 13 z^2 + 36} \, dz$$

(i) **if** C : **the boundary of the rectangular box with vertices** ± 12 **and** $\pm \frac{7}{3} i$ **, and looping in counterclockwise sense .** **(5 marks)**

(ii) **if** C : **the boundary of the sphere** $|z + 5 i| = \frac{9}{4}$ **, and looping in counterclockwise sense .** **(5 marks)**

(b) **Given the following definite integral :**

$$\int_0^{2\pi} \frac{\cos(2\theta)}{3 + \cos(\theta)} d\theta$$

(i) **use** int **command to find its value ,** **(3 marks)**

(ii) **convert it to a complex contour integral , evaluate the value of this contour integral . Compare it with that obtained in (i) .** **(12 marks)**

Question four

(a) **Evaluate by the method of residue integration the following improper integrals :**

$$\int_{-\infty}^{\infty} \frac{\cos(x)}{x^2 + x + 1}\, dx \quad \text{and} \quad \int_{-\infty}^{\infty} \frac{\sin(x)}{x^2 + x + 1}\, dx$$

(15 marks)

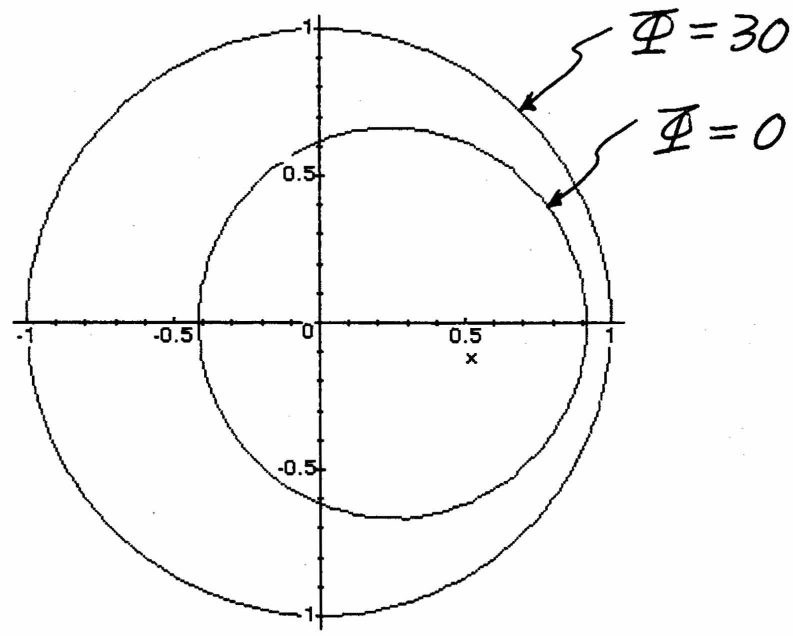
(b) **Find the Cauchy principal value of the following integral :**

$$\int_{-\infty}^{\infty} \frac{1}{x^3 + 4x^2 + 7x + 6}\, dx$$

(10 marks)

Question five

A pair of long, non-coaxial, circular cross-section conductors is statically charged such that the inner conductor (radius of $\frac{2}{3}$ and centred at $\left(x = \frac{1}{4}, y = 0\right)$) is at zero potential, i.e., $\Phi = 0$ volt, while the outer conductor (radius of 1 and centred at origin) is maintained at $\Phi = 30$ volts as shown in the diagram below :



Use the linear fractional transformation of the form $w = \dfrac{z - b}{bz - 1}$ **to transform the above given non-coaxial circles in** $z-plane\ (z = x + i\,y)$ **to two coaxial circles in** $w-plane\ (w = u + i\,v)$ **,**

(a) **find the appropriate value of** b **such that the inner circle of radius** $\frac{2}{3}$ **maps to a coaxial circle of radius** $r_0\ (< 1)$ **. Find also the value of** r_0 **.** **(11 marks)**

Question five (continued)

(b) **since the general solution for coaxial conductors can be written as**

$\Phi = k_1 \ln(|w|) + k_2$ **, determine the values of** k_1 *and* k_2 **from the given boundary conditions .** **(7 marks)**

(c) **plot the equal potential surfaces** $\Phi = 0$, $\Phi = 10$, $\Phi = 20$ **and** $\Phi = 30$ **in** $z-plane$ **and show them in a single display .** **(7 marks)**