

**UNIVERSITY OF SWAZILAND  
FACULTY OF SCIENCE**

**DEPARTMENT OF ELECTRONIC ENGINEERING  
MAIN EXAMINATION, MAY 2007**

**TITLE OF PAPER : DIGITAL COMMUNICATIONS**

**COURSE NUMBER : E530**

**TIME ALLOWED : THREE HOURS**

**INSTRUCTIONS : READ EACH QUESTION CAREFULLY  
ANSWER ANY FOUR OUT OF FIVE  
QUESTIONS. EACH QUESTION CARRIES  
25 MARKS. MARKS FOR EACH SECTION  
ARE SHOWN ON THE RIGHT-HAND  
MARGIN.**

**THIS PAPER HAS 5 PAGES INCLUDING THIS PAGE.**

**THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS  
BEEN GIVEN BY THE INVIGILATOR.**

### USEFUL INFORMATION

$$\begin{aligned}\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B.\end{aligned}$$

$$\begin{aligned}\sin A \sin B &= \frac{1}{2} [\cos(A - B) - \cos(A + B)] \\ \cos A \cos B &= \frac{1}{2} [\cos(A + B) + \cos(A - B)] \\ \sin A \cos B &= \frac{1}{2} [\sin(A + B) + \sin(A - B)]\end{aligned}$$

$$\begin{aligned}\cos^2 A &= \frac{1}{2} [1 + \cos 2A] \\ \sin^2 A &= \frac{1}{2} [1 - \cos 2A]\end{aligned}$$

$$Q(x) = \frac{1}{\sqrt{\pi}} \int_x^\infty e^{-z^2} dz$$

### QUESTION 1

- (a) (i) The communicator needs to transmit a single message, three bits long. The message is convolutionally encoded using the encoder represented by

$$\mathbf{g}_1 = 011, \quad \mathbf{g}_2 = 110.$$

The code symbols are transmitted over a Binary Symmetric Channel with crossover probability  $p = 0.2$ . What is the decoder output when the received sequence is (00 01 11 00 00)? (20 marks)

- (ii) Deduce the original message sequence. (2 marks)

- (b) What is the impulse response of this encoder? (3 marks)

### QUESTION 2

- (a) Bandpass modulation can be defined as the process whereby the amplitude, frequency or phase of an RF carrier, or a combination of them is varied in accordance with the information to be transmitted. The general analytic expression for binary Amplitude Shift Keying (ASK) is

$$S_i(t) = \sqrt{\frac{2E_i(t)}{T_b}} \cos(2\pi f_c t + \theta) \quad 0 \leq t \leq T_b$$

for  $i = 1, \dots, M$ . The phase term  $\theta$  is an arbitrary constant.

- (i) For  $M = 2$ , assuming an Additive White Gaussian Noise (AWGN) model, construct the signal space diagram for ASK (15 marks)
- (ii) Derive the expression for the corresponding average probability of error, assuming that symbols 1 and 0 occur with equal probability. (6 marks)
- (b) The 103/113 FDX modem has a 1170-Hz VCO deviated  $\pm 100$  Hz for the originate mode and a 2125 - Hz VCO deviated  $\pm 100$  Hz for the answer mode. With the aid of a well labeled diagram, show how the modem frequencies can be accommodated in the telephone audio bandpass channel. (4 marks)

### QUESTION 3

(a) Consider a discrete source which emits one of  $X$  discrete symbols from an alphabet  $W = \{S_1, S_2 \dots S_x\}$  in a statistically independent sequence.

Let  $p_1, p_2, \dots p_x$  be the probabilities of occurrence of the  $X$  symbols respectively. In a long message of  $L > X$  symbols, the symbol probabilities change. If one symbol is output each  $T_x$  seconds, derive an expression for the average information per symbol. ( 5 marks )

(b) Suggest a simple design of a correlator receiver which can be used to detect unipolar pulses given a signaling period of 1 s and that the signals form an orthogonal set. ( 11 marks )

(c) A keypad terminal is used to enter alphanumeric data consisting of equiprobable independent sequences, into a computer. The keypad is connected to the computer through a voice grade telephone line with a usable bandwidth of 3000 Hz and an output signal-to-noise ratio of 10 dB. If the terminal has 128 characters, determine

(i) the channel capacity and ( 3 marks )

(ii) the maximum theoretical rate at which data can be transmitted from the terminal to the computer without errors. ( 6 marks )

### QUESTION 4

Four statistically independent symbols,  $a, b, c$  and  $d$ , occurring with probabilities 0.3, 0.2, 0.1 and 0.4 respectively, are to be transmitted. The available communication system can transmit error-free binary symbols at a rate of 1350 bits/s. The source produces the symbols at a rate of 1000 per second.

There are two options available to match the source output to the channel: Huffman coding and straight binary coding.

Which coding method will you choose and why?

Hint: Show all calculations, including code efficiency for each method.

( 25 marks )

### **QUESTION 5**

- (a) An arbitrary matrix  $P$  plays an important part in generation of block codes. It is normally selected to ensure that the generated code has desirable properties like ease of implementation and ability to correct both random and burst errors.

A coded vector is sent over a noisy Gaussian channel and the resulting received vector is observed to be  $[1\ 0\ 1\ 1\ 1\ 0]$ . Will any errors be

correctable if the matrix  $\bar{P} = \begin{bmatrix} 110 \\ 111 \\ 101 \end{bmatrix}$  was used in the decoding process?

Determine the message vector. ( 11 marks )

- (b) A multilevel digital communication system is to operate at a data rate of 9600 bits/sec. If 4 - bit words are encoded into each level for transmission over the channel, what is the minimum required bandwidth for the channel?  
( 4 marks )

- (c) A NRZ polar communication system with equally likely signalling and a peak signal voltage of  $A$  volts is used in the transmission of binary data.

Derive the  $P_e$  expression when a low pass filter is used at the reception.  
( 10 marks )

**THIS IS THE END OF EXAMINATION QUESTIONS**