UNIVERSITY OF SWAZILAND
FACULTY OF SCIENCE
DEPARTMENT OF ELECTRONIC ENGINEERING

SUPPLEMENTARY EXAMINATION 2006/7

TITLE OF PAPER : **ADVANCED CONTROL SYSTEMS**

COURSE NUMBER : **EIN 530**

TIME ALLOWED : THREE HOURS

INSTRUCTIONS : **ANSWER ALL FOUR QUESTIONS**

EACH CARRY 25 MARKS.

MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN

THIS PAPER HAS 6 PAGES, INCLUDING THIS PAGE

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Partial Table of z- and s-Transforms

| | f(t) | F(s) | F(z) | f(kt) |
|---|---|---|---|---|
| 1. | $u(t) = 1$ | $\frac{1}{s}$ | $\frac{z}{z-1}$ | $u(kT)$ |
| 2. | $t$ | $\frac{1}{s^2}$ | $\frac{Tz}{(z-1)^2}$ | $kT$ |
| 3. | $t^n$ | $\frac{n!}{s^{n+1}}$ | $\lim_{a\to 0}(-1)^n \frac{d^n}{da^n}\left[\frac{z}{z-e^{-aT}}\right]$ | $(kT)^n$ |
| 4. | $e^{-at}$ | $\frac{1}{s+a}$ | $\frac{z}{z-e^{-aT}}$ | $e^{-akT}$ |
| 5. | $t^n e^{-at}$ | $\frac{n!}{(s+a)^{n+1}}$ | $(-1)^n \frac{d^n}{da^n}\left[\frac{z}{z-e^{-aT}}\right]$ | $(kT)^n e^{-akT}$ |
| 6. | $\sin \omega t$ | $\frac{\omega}{s^2+\omega^2}$ | $\frac{z \sin \omega T}{z^2 - 2z\cos \omega T + 1}$ | $\sin \omega kT$ |
| 7. | $\cos \omega t$ | $\frac{s}{s^2+\omega^2}$ | $\frac{z(z-\cos \omega T)}{z^2 - 2z\cos \omega T + 1}$ | $\cos \omega kT$ |
| 8. | $e^{-at}\sin \omega t$ | $\frac{\omega}{(s+a)^2+\omega^2}$ | $\frac{ze^{-aT}\sin \omega T}{z^2 - 2ze^{-aT}\cos \omega T + e^{-2aT}}$ | $e^{-akT}\sin \omega kT$ |
| 9. | $e^{-at}\cos \omega t$ | $\frac{s+a}{(s+a)^2+\omega^2}$ | $\frac{z^2 - ze^{-aT}\cos \omega T}{z^2 - 2ze^{-aT}\cos \omega T + e^{-2aT}}$ | $e^{-akT}\cos \omega kT$ |
| 10. | | | $\frac{z}{z+a}$ | $a^K \cos K\pi$ |

z-Transform Theorems

| | Theorem | Name |
|---|---|---|
| 1. | $z\{af(t)\} = aF(z)$ | Linearity theorem |
| 2. | $z\{f_1(t) + f_2(t)\} = F_1(z) + F_2(z)$ | Linearity theorem |
| 3. | $z\{e^{-at}f(t)\} = F(e^{aT}z)$ | Complex differentiation |
| 4. | $z\{f(t-nT)\} = z^{-n}F(z)$ | Real translation |
| 5. | $z\{tf(t)\} = -Tz\frac{dF(z)}{dz}$ | Complex differentiation |
| 6. | $f(0) \doteq \lim_{z\to\infty} F(z)$ | Initial value theorem |
| 7. | $f(\infty) = \lim_{z\to 1}(1-z^{-1})F(z)$ | Final value theorem |

Note: $kT$ may be substituted for $t$ in the table.

Question 1

Pole-placement techniques require that a system be completely controllable and observable to in order to allow the arbitrary placement of closed-loop poles. For a system represented by the following equations

$$\dot{x}_1 = x_2$$

$$\dot{x} = x_3$$

$$\dot{x}_3 = -x_1 - 2x_2 - 3x_3 + u$$

$$y = a_1 x_1 + a_3 x_3$$

Where $\boldsymbol{a}_1$ and $\boldsymbol{a}_3$ are constants.

A) Determine whether this system is controllable [10 marks]

$$Y(s) = \frac{5(s+100)}{s^2 + 60s + 500} R(s).$$

B) if $\boldsymbol{a}_1 = 1$, determine the condition on $\boldsymbol{a}_3$ for this system to be observable. [15 marks]

Question 2

In design of state variable feedback systems the full-state control law plus the observer make up a compensator. Using the requirements stated below, determine

A) the state-feedback matrix K, and [13 marks]

B) the observer gain matrix L for a system represented by [12 marks]

$$\dot{x} = \begin{bmatrix} -5 & -2 \\ 2 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$$

Requirements

For full state-feedback : settling time = **1** second and $\zeta =$ **0. 93**

For the observer : dominant roots are at **s = -3 ± j3**

Question 3

Consider a system as shown below in Figure 3

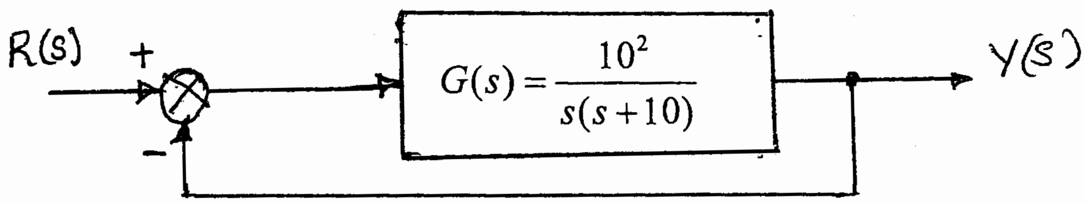


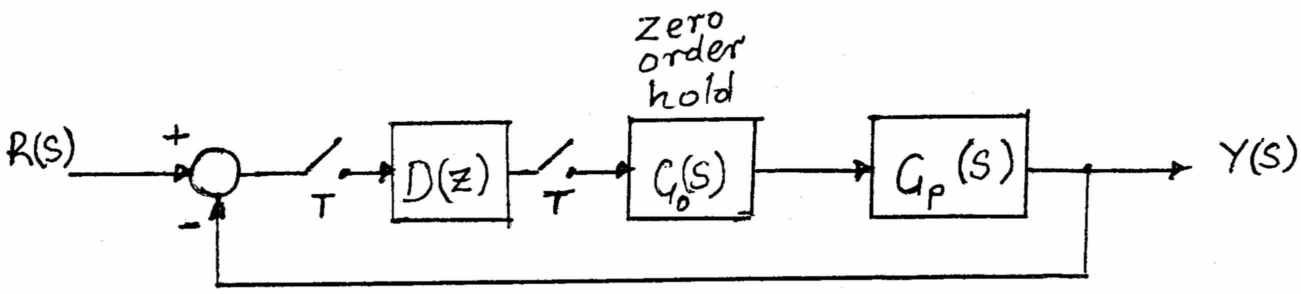

Figure 3

For this system

A) Draw Bode diagrams [14 marks]
B) Determine the phase margin using the Bode diagrams [2 marks]
C) If the desired phase margin is 40° design a compensator to achieve this requirement. [9 marks]

<u>Question 4</u>

Consider the computer-computer-compensated system shown in Figure 4 when T = 1 second.




<u>Figure 4</u>

$$G_p(s) = \frac{K}{s(s+1)} \qquad D(z) = \frac{z-0.3678}{z+a}$$

Select the parameters ***K*** and ***a*** so that the transfer function $T(z) = \frac{Y(z)}{R(z)} = \frac{1}{z}$

[25 marks ]