UNIVERSITY OF SWAZILAND
FACULTY OF SCIENCE
DEPARTMENT OF ELECTRONIC ENGINEERING

MAIN EXAMINATION DECEMBER 2007

TITLE OF PAPER: **CONTROL SYSTEMS**

COURSE CODE: **E430**

TIME ALLOWED: **THREE** HOURS

INSTRUCTIONS:

1. Answer question **one** and any other **three** questions.
2. Question one carries 40 marks.
2. Questions 2, 3, 4, and 5 carry 20 marks each.
3. Marks for different sections are shown in the right-hand margin.

This paper has 7 pages including this page.

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Table of Laplace and Z Transforms

Entry #	Laplace Domain	Time Domain	Z Domain (t=kT)
1	1	$\delta(t)$ unit impulse	1
2	$\frac{1}{s}$	$u(t)$ unit step	$\frac{z}{z-1}$
3	$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$
4	$\frac{1}{s+a}$	e^{-at}	$\frac{z}{z-e^{-aT}}$
5		$b^k \quad \left(b=e^{-aT}\right)$	$\frac{z}{z-b}$
6	$\frac{1}{(s+a)^2}$	te^{-at}	$\frac{Tze^{-aT}}{\left(z-e^{-aT}\right)^2}$
7	$\frac{1}{s(s+a)}$	$\frac{1}{a}\left(1-e^{-at}\right)$	$\frac{z\left(1-e^{-aT}\right)}{a(z-1)\left(z-e^{-aT}\right)}$
8	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$\frac{z\left(e^{-aT}-e^{-bT}\right)}{\left(z-e^{-aT}\right)\left(z-e^{-bT}\right)}$
9	$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}-\frac{e^{-at}}{a(b-a)}-\frac{e^{-bt}}{b(a-b)}$	
10	$\frac{1}{s(s+a)^2}$	$\frac{1}{a^2}\left(1-e^{-at}-ate^{-at}\right)$	
11	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	
12	$\frac{b}{s^2+b^2}$	$\sin(bt)$	$\frac{z\sin(bT)}{z^2-2z\cos(bT)+1}$
13	$\frac{s}{s^2+b^2}$	$\cos(bt)$	$\frac{z(z-\cos(bT))}{z^2-2z\cos(bT)+1}$
14	$\frac{b}{(s+a)^2+b^2}$	$e^{-at}\sin(bt)$	$\frac{ze^{-aT}\sin(bT)}{z^2-2ze^{-aT}\cos(bT)+e^{-2aT}}$
15	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos(bt)$	$\frac{z^2-ze^{-aT}\cos(bT)}{z^2-2ze^{-aT}\cos(bT)+e^{-2aT}}$
16	$\frac{Bs+C}{(s+a)^2+\omega_n^2}$	$e^{-at}\left[B\cos(\omega_n t)+\frac{C-aB}{\omega_n}\sin(\omega_n t)\right]$	
17		$\sqrt{\frac{a^2d^2+b^2-2abc}{d^2-c^2}}\,d^n\cos(\beta n+\gamma)$ $\beta=\cos^{-1}\left(-\frac{c}{d}\right),\quad \gamma=\tan^{-1}\left(\frac{ac-b}{a\sqrt{d^2-c^2}}\right)$	$\frac{az^2+bz}{z^2+2cz+d^2},\quad d>0$
Prototype Second Order System (ζ<1, underdampded)			
18	$\frac{\omega_n^2}{s\left(s^2+2\zeta\omega_n s+\omega_n^2\right)}$	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\left(\omega_n\sqrt{1-\zeta^2}\,t+\cos^{-1}(\zeta)\right)$	$\frac{z}{z-1}-\frac{1}{\sqrt{1-\zeta^2}}\frac{z^2\sqrt{1-\zeta^2}+z\sin\left(\omega_n\sqrt{1-\zeta^2}\,T-\cos^{-1}(\zeta)\right)e^{-\zeta\omega_n T}}{z^2-2ze^{-\zeta\omega_n T}\cos\left(\omega_n\sqrt{1-\zeta^2}\,T\right)+e^{-2\zeta\omega_n T}}$
19	$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\left(\omega_n\sqrt{1-\zeta^2}\,t\right)$	$\frac{\omega_n}{\sqrt{1-\zeta^2}}\frac{ze^{-\zeta\omega_n T}\sin\left(\omega_n\sqrt{1-\zeta^2}\,T\right)}{z^2-2ze^{-\zeta\omega_n T}\cos\left(\omega_n\sqrt{1-\zeta^2}\,T\right)+e^{-2\zeta\omega_n T}}$
20	$\frac{s\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{-\omega_n^2}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\left(\omega_n\sqrt{1-\zeta^2}\,t-\cos^{-1}(\zeta)\right)$	$\frac{-\omega_n^2}{\sqrt{1-\zeta^2}}\frac{-z^2\sqrt{1-\zeta^2}+z\sin\left(\omega_n\sqrt{1-\zeta^2}\,T+\cos^{-1}(\zeta)\right)e^{-\zeta\omega_n T}}{z^2-2ze^{-\zeta\omega_n T}\cos\left(\omega_n\sqrt{1-\zeta^2}\,T\right)+e^{-2\zeta\omega_n T}}$

Question 1

(a) A home shower with separate hot and cold water valves is an example of a two input control system. A desired temperature and a desired flow of water is required. Sketch a block diagram of this closed-loop system. [10 marks]

(b) A feedback system is shown in Figure 1. The input $R(s)$ is a unit step.

(i) Determine the steady state error when $G_p(s) = 1$. [7 marks]

(ii) Select an appropriate value for $G_p(s)$ so that the steady-state error is equal to zero. [9 marks]

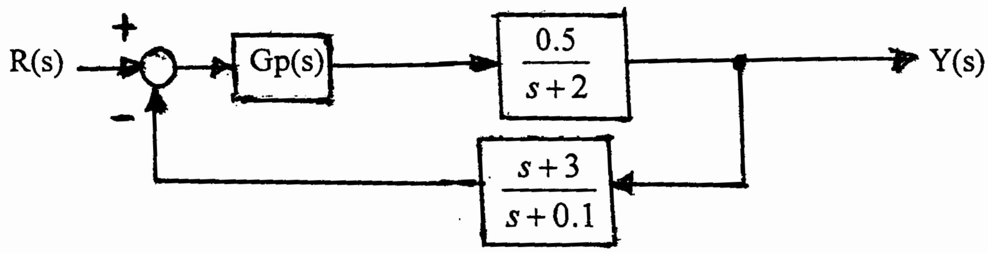

Figure 1

(c) A feedback control system has a characteristic equation

$$s^3 + (1+K)s^2 + 10s + (50-5K) = 0$$

(i) Determine the range of K for which the system is stable.

(ii) At what frequency does the system oscillate.

[9 marks]

(d) (i) What is the difference between a controller and a compensator? [2 marks]

(ii)Give three examples of classification of industrial control action. [3 marks]

Question 2

A mechanical system representing the vibration of a machine with unbalanced load is shown in Figure 2.
Obtain the characteristic equation of this system when the input is *F(t)*.

[20 marks]

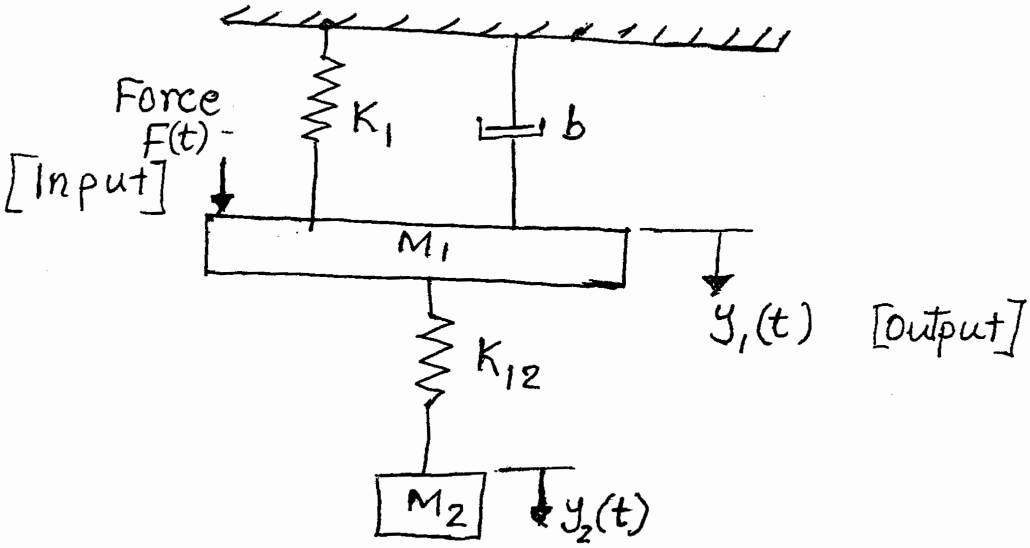

Figure 2

Question 3

A system shown in Figure 3 has a plant transfer function $G_p(s) = \dfrac{100}{s^2 + 100}$

(a) Determine the impulse transfer function $G_p(z)$ [10 marks]
(b) Determine whether this system is stable. [3 marks]
(c) Determine the magnitude of the impulse responses for the first four samples. [7 marks]

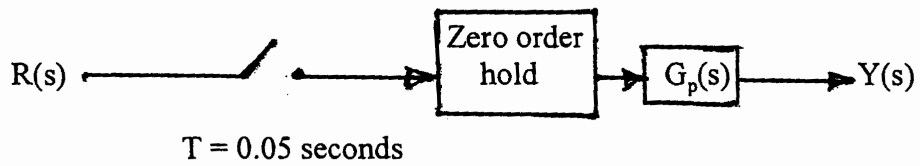

Figure 3

Question 4

Draw to scale the Bode Magnitude and Phase plots of the transfer function

$$G(s) = \frac{(1+\frac{s}{1500})^2(1+\frac{s}{10000})}{10(1+\frac{s}{500})^2(1+\frac{s}{100000})}$$

[20 marks]

Question 5

Draw the root locus for a unity feedback system with

$$G(s) = \frac{k(s+10)}{s(s+1)(s+20)}$$

and indicate on the plot the roots when k =100. [20 marks]