

**UNIVERSITY OF SWAZILAND  
FACULTY OF SCIENCE  
DEPARTMENT OF ELECTRONIC ENGINEERING**

**SUPPLEMENTARY EXAMINATION JULY 2008**

**TITLE OF PAPER: CONTROL SYSTEMS**

**COURSE CODE: E430**

**TIME ALLOWED: THREE HOURS**

**INSTRUCTIONS:**

1. Answer all four questions.
2. Each Question carries 25 marks.
3. Marks for different sections are shown in the right-hand margin

This paper has 6 pages including this page.

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BY THE INVIGILATOR.**

Partial Table of z- and s-Transforms

$f(t)$	$F(s)$	$F(z)$	$f(kT)$
1. $u(t)$	$\frac{1}{s}$	$\frac{z}{z-1}$	$u(kT)$
2. $t$	$\frac{1}{s^2}$	$\frac{Tz}{(z-1)^2}$	$kT$
3. $t^n$	$\frac{n!}{s^{n+1}}$	$\lim_{a \rightarrow 0} (-1)^n \frac{d^n}{da^n} \left[ \frac{z}{z - e^{-aT}} \right]$	$(kT)^n$
4. $e^{-at}$	$\frac{1}{s+a}$	$\frac{z}{z - e^{-aT}}$	$e^{-akT}$
5. $t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$	$(-1)^n \frac{d^n}{da^n} \left[ \frac{z}{z - e^{-aT}} \right]$	$(kT)^n e^{-akT}$
6. $\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$	$\frac{z \sin \omega T}{z^2 - 2z \cos \omega T + 1}$	$\sin \omega kT$
7. $\cos \omega t$	$\frac{s}{s^2 + \omega^2}$	$\frac{z(z - \cos \omega T)}{z^2 - 2z \cos \omega T + 1}$	$\cos \omega kT$
8. $e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$	$\frac{ze^{-aT} \sin \omega T}{z^2 - 2ze^{-aT} \cos \omega T + e^{-2aT}}$	$e^{-akT} \sin \omega kT$
9. $e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$	$\frac{z^2 - ze^{-aT} \cos \omega T}{z^2 - 2ze^{-aT} \cos \omega T + e^{-2aT}}$	$e^{-akT} \cos \omega kT$
10.		$\frac{z}{z+a}$	$a^k \cos k\pi$

z-Transform Theorems

Theorem	Name
1. $z\{af(t)\} = aF(z)$	Linearity theorem
2. $z\{f_1(t) + f_2(t)\} = F_1(z) + F_2(z)$	Linearity theorem
3. $z\{e^{-at} f(t)\} = F(e^{aT} z)$	Complex differentiation
4. $z\{f(t - nT)\} = z^{-n} F(z)$	Real translation
5. $z\{t f(t)\} = -T z \frac{dF(z)}{dz}$	Complex differentiation
6. $f(0) = \lim_{z \rightarrow \infty} F(z)$	Initial value theorem
7. $f(\infty) = \lim_{z \rightarrow 1} (1 - z^{-1}) F(z)$	Final value theorem

Note:  $kT$  may be substituted for  $t$  in the table.

**Question 1**

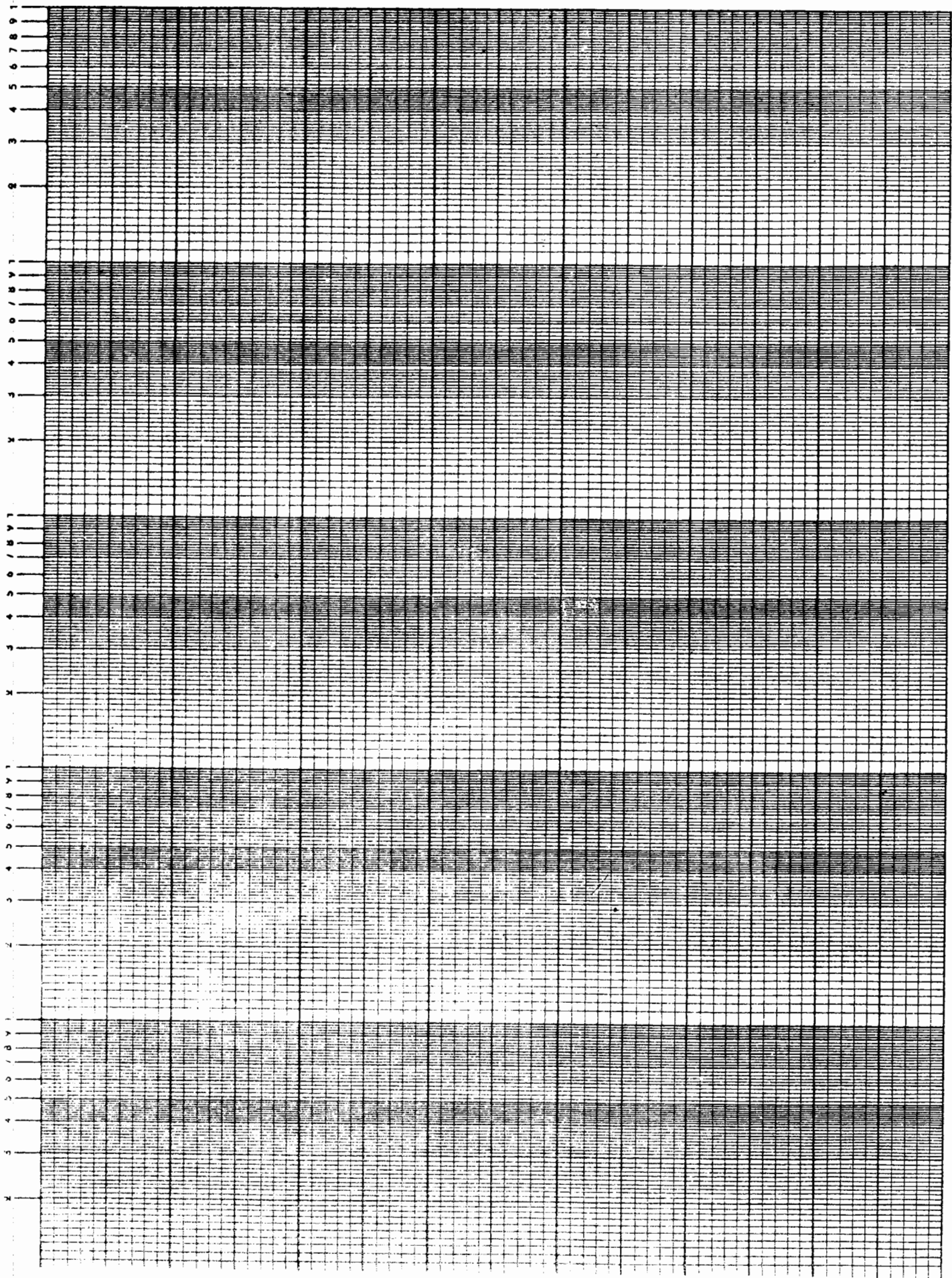
A system is given by the following transfer function

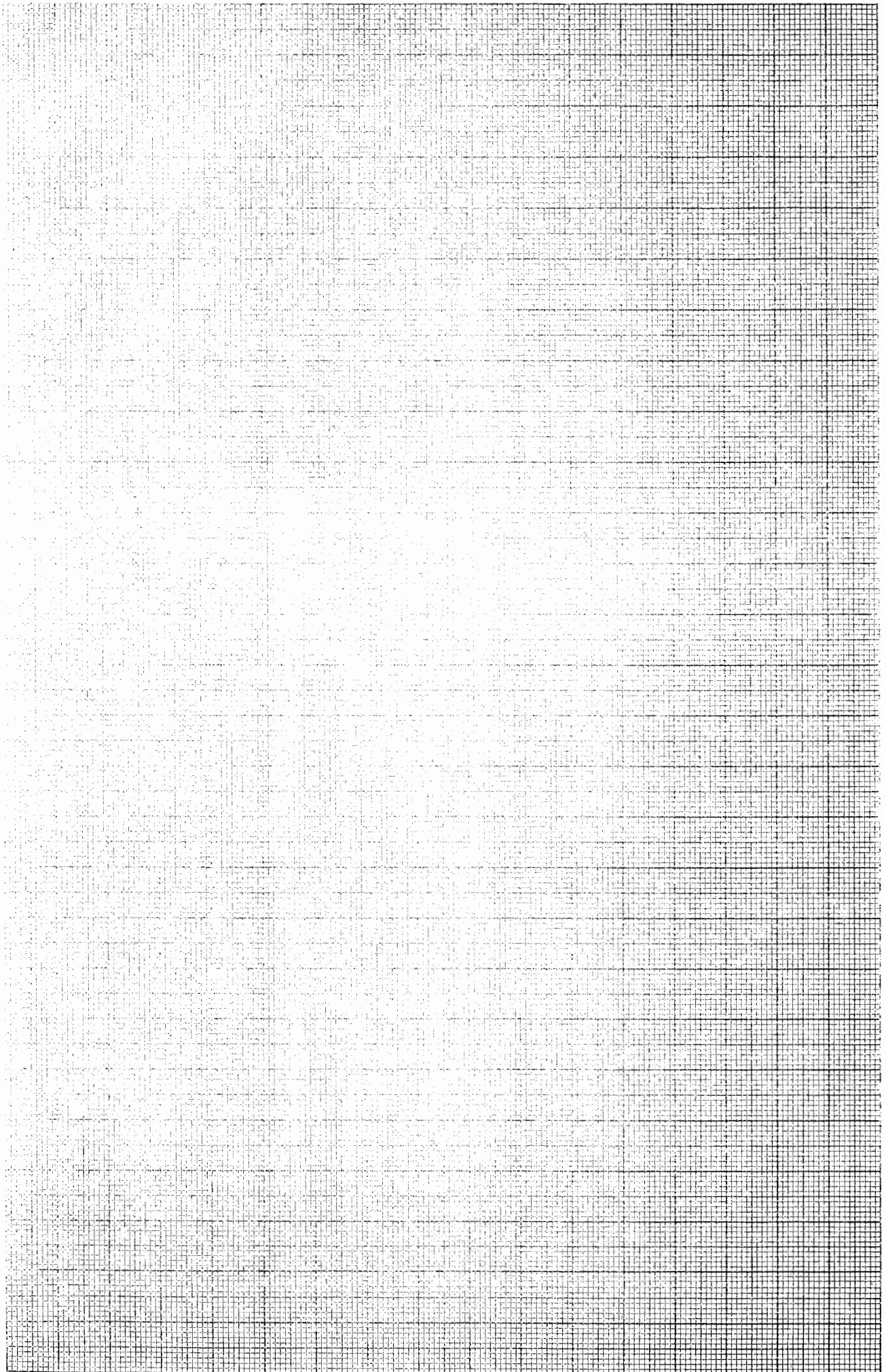
$$\frac{Y(s)}{R(s)} = \frac{10(s+2)}{(s+0.2)(s^2+5s+100)}$$

A) Determine response  $y(t)$  when the input  $r(t)$  is a unit step. [10 marks]

B) Draw the Bode diagram (Magnitude only) for this system. [15 marks]

SEMI-LOG PAPER (5 CYCLES x 1/10")

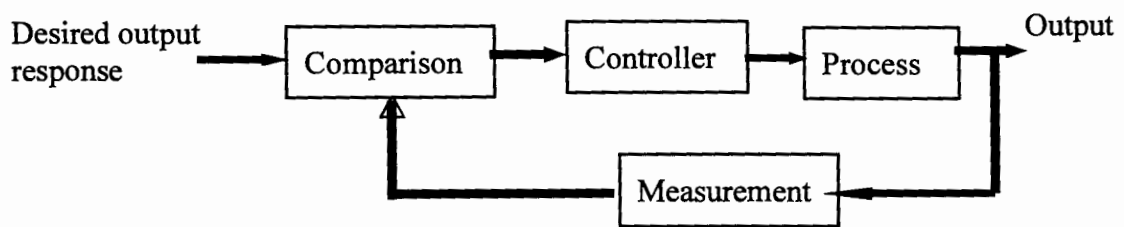




**Question 2**

- A) The student-teacher learning process is inherently a feedback process intended to reduce the system error to a minimum. In this process knowledge is transferred. With the aid of Figure 2 construct a feedback model of the learning process and identify each block of the system.

[6 marks]



**Figure 2**

- B) The characteristic equation of a sampled system is  $z^2 + (K - 1.5)z + 0.5 = 0$   
Find the range of the gain K so that the system is stable. [8 marks]
- C) (i) Determine  $y(k)$  for  $k = 0$  to 3 when  $Y(z) = \frac{z + 2}{z^2 - 1}$  [6 marks ]  
(ii) Determine the closed form solution for  $y(k)$  as a function of k [5 marks]

**Question 3**

A) Represent the following transfer function in state space, use any format.

$$T(s) = \frac{s^2 + 7s + 10}{(s + 1)(s^2 + 5s + 9)} \quad [10 \text{ marks}]$$

B) Find the damping ratio, natural frequency, percent overshoot and peak time for the system shown in Figure 3. [ 15 marks ]

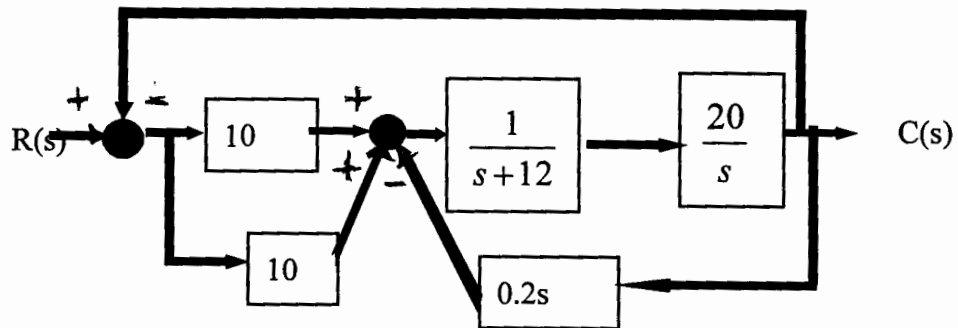


Figure 3

**Question 4**

A unity feedback system has an open-loop transfer function

$$KG(s) = K \frac{s + 10}{(s + 2)(s + 4)(s + 8)}$$

- A) Determine the breakaway point and the value of K at this point [5 marks]
- B) Is there a value of K that will make this system critically stable? Show. [5 marks]
- C) Determine the centre of asymptotes and the angle of the asymptotes [2 marks]
- D) Determine the roots when K=1 and K=10 [4 marks]
- E) Sketch the root locus [9 marks]