UNIVERSITY OF SWAZILAND
FACULTY OF SCIENCE
DEPARTMENT OF ELECTRONIC ENGINEERING

MAIN EXAMINATION 2007/08

TITLE OF PAPER : **SIGNALS II**

COURSE NUMBER : **E462**

TIME ALLOWED : **THREE (3) HOURS**

INSTRUCTIONS : **ANSWER ANY FOUR OUT OF THE FIVE QUESTIONS**

EACH QUESTION CARRIES 25 MARKS

MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN

THIS PAPER CONTAINS 8 PAGES INCLUDING THIS PAGE

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QUESTION 0NE

(a) Given that the probability density function for wait in line at a counter in the production of Op-amps is

$$x(t)=\begin{cases}0.1e^{-t/10} & if\ t\ge 0\\ 0 & if\ t<0\end{cases},$$

where t is the number of minutes spent waiting in line.

(i) Determine the probability that an Op-amp will wait in line for at least 6 minutes. [3]

(ii) Determine the mean wait time. [5]

(b) Suppose that $x(t)$ has the Fourier transform $u(1-\omega^2)$. Use the appropriate Fourier transform pairs/properties to find the Fourier Transform of the following signal;

$$x(5-3t).$$ [6]

(c) Calculate the linear convolution of the following discrete-time sequences and express your result, *y[n],* in terms of the delta function;

$$x[n]=\partial[n+1]-2\partial[n]+3\partial[n-1],$$
$$h[n]=\partial[n]+4\partial[n-1]-2\partial[n-2]+\partial[n-3].$$ [6]

(d) Find the inverse Fourier transform of the following signal; [5]

$$X(\omega)=\frac{1}{1+j\omega}\cos(2\omega)e^{-j5\omega}.$$

QUESTION TWO

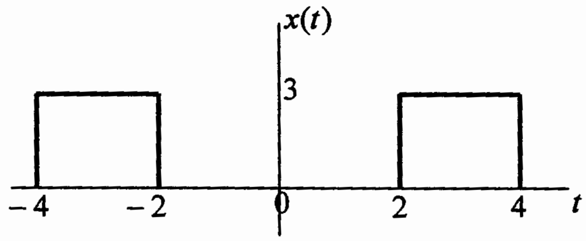
(a) Find the response of a linear system to an input of $x(t) = u(t) - u(t-4)$ if the impulse response $h(t) = r(t)$.

[note: $r(t)$ is the ramp function and $u(t)$ is the unit step] [8]

(b) Either by calculation and/or the use of Fourier Transform tables, find the Fourier Transform, $X(\omega)$, of the following signals:

(i) $x(t) = e^{-2t}\cos(4t)u(t)$ [5]

(ii)



[8]

*[note: express your answer in terms of the **sinc** function]*

(c) An N-sample signal $x[n]$ has the DFT $X[k]$. Use the appropriate DFT property(ies) to write down expressions for the DFTs of the signals;

(i) $x[n]x[n-1]$. [2]

(ii) $2x[n] + x[n+1]$. [2]

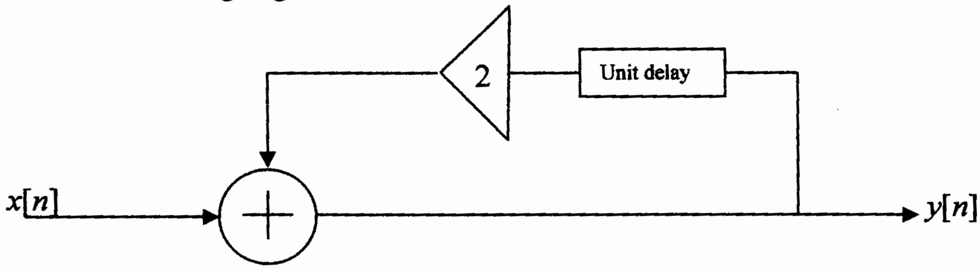
QUESTION THREE

Note: the term z-transform generally means the unilateral z-transform. Hence there is no need for ROC specification

(a) Evaluate the *Z*-Transform of the following sequence:

$$x[n] = 2(0.8)^n u[n].$$ [4]

(b) For the following digital filter structure

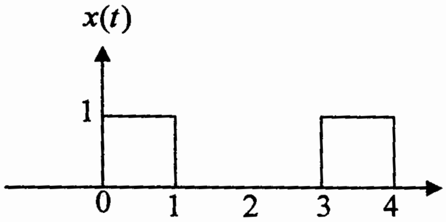


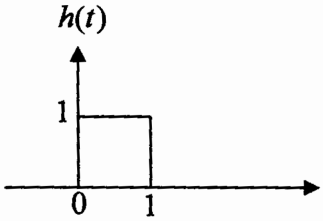
(i) Find the transfer function. [3]

(ii) Find the first four values of the output sequence *y[n]* corresponding to to the input $x[n] = \partial[n]$. [4]

(iii) Find the first four values of the output sequence *y[n]* corresponding to the input $x[n] = u[n]$. [4]

(c) Find and sketch $y(t) = x(t) * h(t)$ for the following functions; [10]





QUESTION FOUR

Note: the term z-transform generally means the unilateral z-transform. Hence there is no need for ROC specification

(a) Determine if the following function is a probability density function;

$$4rect(4(t-2)).$$ [3]

(b) Determine the energy spectral density of the following function;

$$-2rect(4t+2).$$ [6]

(c) Compute the DFT of the following 4-point causal sequence; [10]

$$x[n]=(-1)^n.$$

(d) Find the inverse Z-transform for a causal stable system defined by

$$H(z)=\frac{2-\frac{5}{6}z^{-1}}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}}.$$ [4]

(e) Find the z-transform of the following signal; [2]

$$x[n]=u[n]-u[n-4].$$

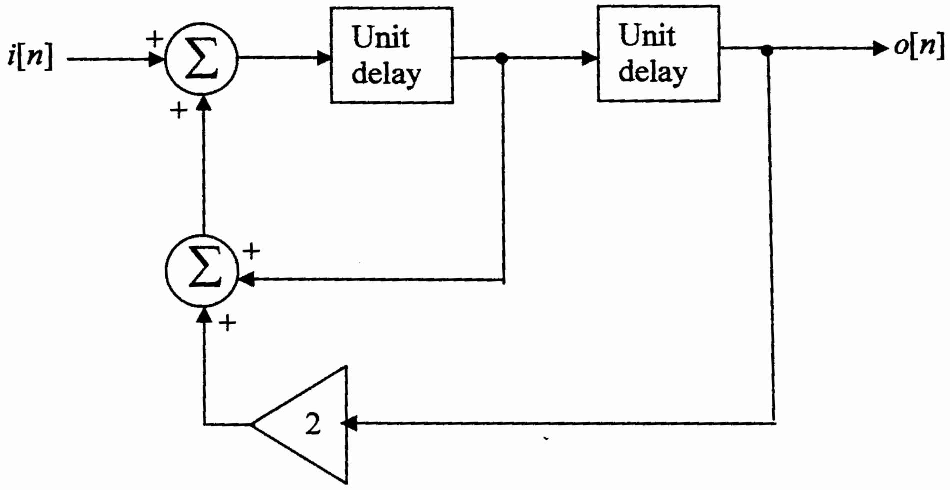
QUESTION FIVE

(a) Explain the following terms when used in association with filters:

(i) cut-off frequencies [2]

(ii) 3 dB bandwidth [3]

(b) (i) Find the transfer function of the following digital filter [5]



(ii) Is it a FIR or IIR filter? Justify your answer. [2]

(c) Determine the bandwidth of an RC low-pass filter with R=10kΩ and C=0.1μF. [2]

(d) Find the impulse response for the following discrete-time system; [6]

$$y[n]+0.2y[n-1]=x[n]-x[n-1].$$

(e) Given the following discrete-time signals evaluate and sketch $y[n]=x[n]*h[n]$

$$x[n]=\begin{cases}-1, & n=-2\\ -1, & n=-1\\ 1, & n=0\\ 1, & n=1\\ 1, & n=2\\ 0, & otherwise\end{cases} \qquad h[n]=\begin{cases}1, & n=0\\ 1, & n=1\\ 1, & n=2\\ 0. & otherwise\end{cases}$$

[5]

TABLE 1: PROPERTIES OF THE FOURIER TRANSFORM

Property	Signal	Fourier transform
	$x(t)$	$X(\omega)$
Linearity	$a_1x_1(t)+a_2x_2(t)$	$a_1X_1(\omega)+a_2X_2(\omega)$
Time shifting	$x(t-t_0)$	$e^{-j\omega t_0}X(\omega)$
Frequency shifting	$e^{j\omega_0 t}x(t)$	$X(\omega-\omega_0)$
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$
Time reversal	$x(-t)$	$X(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Frequency differentiation	$(-jt)x(t)$	$\frac{dX(\omega)}{d\omega}$
Time differentiation	$\frac{dx(t)}{dt}$	$j\omega X(\omega)$
Integration	$\int_{-\infty}^{t}x(\tau)d\tau$	$\pi X(0)\delta(\omega)+\frac{1}{j\omega}X(\omega)$
Convolution	$x_1(t)*x_2(t)$	$X_1(\omega)X_2(\omega)$
Multiplication	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(\omega)*X_2(\omega)$

TABLE 2: COMMON FOURIER TRANSFORM PAIRS

Continuous time function $x(t)$	Fourier transform $X(\omega)$
1	$2\pi\delta(\omega)$
$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$
$\delta(t)$	1
$\delta(t-t_0)$	$e^{-j\omega t_0}$
$rect\left(t/\tau\right)$	$\frac{2\sin\left(\omega\tau/2\right)}{\omega}=\tau\,\mathrm{sin}\,c\left(\omega\tau/2\right)$
$\frac{\sin(\omega_0 t)}{\pi t}=\frac{\omega_0}{\pi}\mathrm{sin}\,c(\omega_0 t)$	$rect\left(\frac{\omega}{2\omega_0}\right)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
$\cos(\omega_0 t)$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$
$\sin(\omega_0 t)$	$\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$
$\cos(\omega_0 t)u(t)$	$\frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\frac{j\omega}{\omega_0^2-\omega^2}$
$\sin(\omega_0 t)u(t)$	$\frac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$
$e^{-at}u(t)$	$\frac{1}{a+j\omega}$
$te^{-at}u(t)$	$\frac{1}{(a+j\omega)^2}$
$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2+\omega^2}$
$\lvert t\rvert e^{-a\lvert t\rvert}$	$\frac{2(a^2-\omega^2)}{a^2+\omega^2}$

SOME COMMON Z-TRANSFORM PAIRS

$x[n]$	$X(z)$	ROC
$\delta[n]$	1	$all\ z$
$u[n]$	$\frac{1}{1-z^{-1}}, \frac{z}{z-1}$	$\lvert z\rvert > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}, \frac{z}{z-1}$	$\lvert z\rvert < 1$
$\delta[n-m]$	z^{-m}	All z except 0 if $(m>0)$ or ∞ if $(m<0)$
$a^n u[n]$	$\frac{1}{1-az^{-1}}, \frac{z}{z-a}$	$\lvert z\rvert > \lvert a\rvert$
$a^{n-1}u[n-1]$	$\frac{1}{z-a}$	
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}, \frac{z}{z-a}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}, \frac{az}{(z-a)^2}$	$\lvert z\rvert > \lvert a\rvert$
$(\cos\Omega_0 n)u[n]$	$\frac{z^2-(\cos\Omega_0)z}{z^2-(2\cos\Omega_0)z+1}$	$\lvert z\rvert > 1$
$(\sin\Omega_0 n)u[n]$	$\frac{(\sin\Omega_0)z}{z^2-(2\cos\Omega_0)z+1}$	$\lvert z\rvert > 1$
$(r^n\cos\Omega_0 n)u[n]$	$\frac{z^2-(r\cos\Omega_0)z}{z^2-(2r\cos\Omega_0)z+r^2}$	$\lvert z\rvert > r$
$(r^n\sin\Omega_0 n)u[n]$	$\frac{(r\sin\Omega_0)z}{z^2-(2r\cos\Omega_0)z+r^2}$	$\lvert z\rvert > r$

SOME PROPERTIES OF THE DFT

1. Linearity

$$a_1x_1[n]+a_2x_2[n] \leftrightarrow a_1X_1[k]+a_2X_2[k]$$

2. Time-shifting

$$x[n-n_0] \leftrightarrow X[k]e^{-j\frac{2\pi kn_0}{N}} = X[k]W_N^{kn_0}$$

3. Modulation/Multiplication

$$x_1[n]x_2[n] \leftrightarrow \frac{1}{N}\sum_{m=0}^{N-1} X_1[m]X_2[k-m]$$

4. Frequency Shifting

$$W_N^{-kn_0}x[n] \leftrightarrow X[k-k_0]$$

5. Time reversal

$$x[-n] \leftrightarrow X[-k]$$

6. Convolution

$$\sum_{m=0}^{N-1} x_1[n]x_2[m-n] \leftrightarrow X_1[k]X_2[k]$$