UNIVERSITY OF SWAZILAND
FACULTY OF SCIENCE
DEPARTMENT OF ELECTRONIC ENGINEERING

SUPPLEMENTARY EXAMINATION 2007/08

TITLE OF PAPER : SIGNALS II

COURSE CODE : E462

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS : ANSWER ANY FOUR OUT OF THE FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN

THIS PAPER HAS 8 PAGES, INCLUDING THIS PAGE

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QUESTION ONE

(a) An electrical network has the following impulse response

$$h(t) = \begin{cases} 5\times10^3 e^{-t/0.001}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

(i) Find the spectrum of the impulse response [4]

(ii) Find the network's amplitude response to an input sine wave of amplitude 2 volts and frequency 200 Hz [6]

(b) Find and sketch (and label) the Fourier transform of the following signal

$$f(t) = \begin{cases} 1 & |t| < a \\ 0 & |t| > a \end{cases}$$ [7]

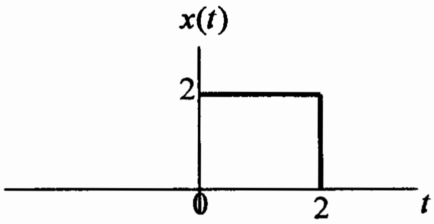
(c) Calculate the output $y[n]$ of a digital system whose input is $x[n] = [0, 1, -1, 1, 1]$ and impulse response is $h[n] = [3, 2, 4, 1]$ by using circular convolution

[8]

QUESTION TWO

(a) By calculation and/or use of Fourier transform tables, find the Fourier transform, $X(\omega)$, of the following signals

(i)



[*note: express your answer in terms of the* ***sinc*** *function*] [7]

(ii) $x(t) = e^{-3t}\cos(10t)u(t)$ [7]

(b) (i) Define statistically independence when used in association with probability theory.
[2]

(ii) Two cards are dealt one at a time, face up, from a shuffled pack of 52 cards. Show that these two cards are not statistically independent
[4]

(c) What is the difference between cross-correlation and auto-correlation. Give any one typical application where correlation may be used
[5]

QUESTION THREE

(a) Given that

$$f(x)=\begin{cases}\dfrac{x^3}{5000}(10-x) & \text{if } 0\le x\le 10\\ 0 & \text{elsewhere}\end{cases}$$

(i) Show that $f(x)$ is a probability density function [3]

(ii) Determine $P(X\ge 6)$ [4]

(b) A digital processor is described by a unit impulse response, $h[n]=[0\ \ 1\ \ 2\ \ 3\ \ 4]$. If the input sequence is $x[n]=[1\ \ 4\ \ 8\ \ 2]$, determine the output, y[n]

[5]

(c) Given the following probability density function

$$v(t)=\begin{cases}\dfrac{1}{2}t & 0<t<2\\ 0 & \text{otherwise}\end{cases}$$

Calculate the following,

(i) mean [3]

(ii) variance [5]

(d) Sketch and label the Fourier transform of the impulse delta function [2]

(e) Verify the time-shifting property of the Fourier transform [3]

QUESTION FOUR

(a) Determine $f(2 < X < 3)$given that the probability density function of the random variable X is given by:

$$f(x) = \begin{cases} \frac{1}{8}(x+1) & for \quad 2 \le x < 4 \\ 0 & elsewhere \end{cases}$$ [4]

(b) Find the Z-transform of the following signal:

$$x[n] = 0.5^n u[n]$$ [4]

(c) Find the inverse Z-transform of the following signal

$$X(z) = \frac{(z-1)(z+0.8)}{(z-0.5)(z+0.2)}$$ [5]

(d) Find the Fourier transform of the following signal at frequency $f = 3\,Hz$:

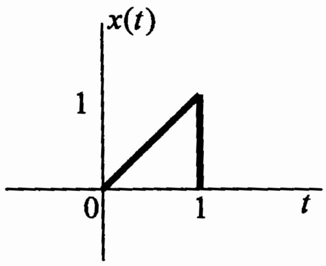
$$x(t) = \begin{cases} t & 0 < t \le 1 \\ 0 & otherwise \end{cases}$$ [8]

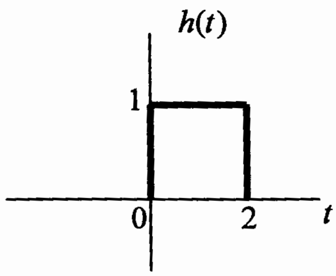
(e) Using the tables of common pairs and properties of the Fourier transform find the Fourier transform of the signal

$$te^{-2|t|}$$ [4]

QUESTION FIVE

(a) Given the following information, determine $y(t) = x(t) * h(t)$ [10]





(b) (i) Determine the time series, $x[n]$, of the following DFT components,
X[k] = [2, 1+j, 0, 1-j] [8]

(ii) Verify that the 12th DFT component is equal to the 0th component in the above components [2]

(c) Perform the cross-correlation $R_{xy}[k]$ of the following periodic sequences
$x[n] = [2, 1, 3, 0]$ and $h[n] = [3, 2, 4, 3]$ [5]

TABLE 1: PROPERTIES OF THE FOURIER TRANSFORM

Property	Signal	Fourier transform
	$x(t)$	$X(\omega)$
Linearity	$a_1x_1(t)+a_2x_2(t)$	$a_1X_1(\omega)+a_2X_2(\omega)$
Time shifting	$x(t-t_0)$	$e^{-j\omega t_0}X(\omega)$
Frequency shifting	$e^{j\omega_0 t}x(t)$	$X(\omega-\omega_0)$
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$
Time reversal	$x(-t)$	$X(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Frequency differentiation	$(-jt)x(t)$	$\frac{dX(\omega)}{d\omega}$
Time differentiation	$\frac{dx(t)}{dt}$	$j\omega X(\omega)$
Integration	$\int_{-\infty}^{t}x(\tau)d\tau$	$\pi X(0)\delta(\omega)+\frac{1}{j\omega}X(\omega)$
Convolution	$x_1(t)*x_2(t)$	$X_1(\omega)X_2(\omega)$
Multiplication	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(\omega)*X_2(\omega)$

TABLE 2: COMMON FOURIER TRANSFORM PAIRS

Continuous time function $x(t)$	Fourier transform $X(\omega)$
1	$2\pi\delta(\omega)$
$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$
$\delta(t)$	1
$\delta(t-t_0)$	$e^{-j\omega t_0}$
$rect\left(t/\tau\right)$	$\frac{2\sin\left(\omega\tau/2\right)}{\omega}=\tau\,\mathrm{sinc}\left(\omega\tau/2\right)$
$\frac{\sin(\omega_0 t)}{\pi t}=\frac{\omega_0}{\pi}\mathrm{sinc}(\omega_0 t)$	$rect\left(\frac{\omega}{2\omega_0}\right)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
$\cos(\omega_0 t)$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$
$\sin(\omega_0 t)$	$\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$
$\cos(\omega_0 t)u(t)$	$\frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\frac{j\omega}{\omega_0^2-\omega^2}$
$\sin(\omega_0 t)u(t)$	$\frac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$
$e^{-at}u(t)$	$\frac{1}{a+j\omega}$
$te^{-at}u(t)$	$\frac{1}{(a+j\omega)^2}$
$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2+\omega^2}$
$\lvert t\rvert e^{-a\lvert t\rvert}$	$\frac{2(a^2-\omega^2)}{a^2+\omega^2}$

TABLE 3: SOME COMMON Z-TRANSFORM PAIRS

$x[n]$	$X(z)$	ROC
$\delta[n]$	1	*all* z
$u[n]$	$\frac{1}{1-z^{-1}}, \frac{z}{z-1}$	$\lvert z\rvert > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}, \frac{z}{z-1}$	$\lvert z\rvert < 1$
$\delta[n-m]$	z^{-m}	*All z except* 0 *if* $(m>0)$ *or* ∞ *if* $(m<0)$
$a^n u[n]$	$\frac{1}{1-az^{-1}}, \frac{z}{z-a}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}, \frac{z}{z-a}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}, \frac{az}{(z-a)^2}$	$\lvert z\rvert > \lvert a\rvert$
$(\cos\Omega_0 n)u[n]$	$\frac{z^2-(\cos\Omega_0)z}{z^2-(2\cos\Omega_0)z+1}$	$\lvert z\rvert > 1$
$(\sin\Omega_0 n)u[n]$	$\frac{(\sin\Omega_0)z}{z^2-(2\cos\Omega_0)z+1}$	$\lvert z\rvert > 1$
$(r^n\cos\Omega_0 n)u[n]$	$\frac{z^2-(r\cos\Omega_0)z}{z^2-(2r\cos\Omega_0)z+r^2}$	$\lvert z\rvert > r$
$(r^n\sin\Omega_0 n)u[n]$	$\frac{(r\sin\Omega_0)z}{z^2-(2r\cos\Omega_0)z+r^2}$	$\lvert z\rvert > r$
$\begin{cases} a^n & 0\le n\le N-1 \\ 0 & \textit{otherwise} \end{cases}$	$\frac{1-a^N z^{-N}}{1-az^{-1}}$	$\lvert z\rvert > 0$

SOME PROPERTIES OF THE DFT

1. Linearity

$$a_1x_1[n]+a_2x_2[n] \leftrightarrow a_1X_1[k]+a_2X_2[k]$$

2. Time-shifting

$$x[n-n_0] \leftrightarrow X[k]e^{-j\frac{2\pi kn_0}{N}} = X[k]W_N^{kn_0}$$

3. Modulation/Multiplication

$$x_1[n]x_2[n] \leftrightarrow \frac{1}{N}\sum_{m=0}^{N-1} X_1[m]X_2[k-m]$$

4. Frequency Shifting

$$W_N^{-k_0 n}x[n] \leftrightarrow X[k-k_0]$$

5. Time reversal

$$x[-n] \leftrightarrow X[-k]$$

6. Convolution

$$\sum_{m=0}^{N-1} x_1[n]x_2[m-n] \leftrightarrow X_1[k]X_2[k]$$