UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRONIC ENGINEERING

SUPPLEMENTARY EXAMINATION 2007/2008

TITLE OF PAPER	:	COMPLEX VARIABLES
COURSE NUMBER	:	E471
TIME ALLOWED	:	THREE HOURS
INSTRUCTIONS	:	ANSWER ANY FOUR OUT OF FIVE QUESTIONS. EACH QUESTION CARRIES 25 MARKS. MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.

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E471 COMPLEX VARIABLES

Question one

(a) **Convert the given complex number** $\left(\frac{9-6i}{7+2i}\right)^{3}$ **into its polar form as well as its** $x+iy$ **form .** **(3 marks)**

(b) **Given the following complex function** $f(z)=\frac{e^{2z}-z}{z^{3}+3}$ **where** $z=x+iy$,

(i) **find its** $u(x,y)$ *and* $v(x,y)$, **(4 marks)**

(ii) **check for its analyticity ,** **(5 marks)**

(iii) **plot the mapped image of** $u(x,y)=5$ **and** $u(x,y)=10$ **curves onto the** $z-plane$ **and show them in one display in** $z-plane$ **for**

$x=0$ *to* $+2$ **and** $y=-5$ *to* $+5$. **(5 marks)**

(Note : $f(z)=u(x,y)+iv(x,y)$)

(c) **Determine the value of** a **and** b **such that** $u(x,y)=a\,x^{3}+b\,x\,y$ **is a harmonic and then find its conjugate harmonic** $v(x,y)$. **(8 marks)**

Question two

(a) Evaluate the value of the following complex line integral $\int_C \left(\sin(2z) + z^2\right) dz$ if

(i) C : the shortest path from $-5-5i$ to $5+5i$, (6 marks)

(ii) C : the counter clockwise circular path from $-5-5i$ to $5+5i$ with the centre of the circle at the origin.

Compare the answer here with that obtained in (a)(i) and make brief comment. (7 marks)

(b) Given the following complex function $f(z)$ as :

$$f(z) = \frac{7i}{z+3} + \frac{6}{z-4i}$$

(i) find its convergent series expansion about $z = -6-4i$ for all the values of z in the domain of $5 < \left|z - (-6-4i)\right| < 10$, (7 marks)

(ii) find its convergent series expansion about $z = 4$ for all the values of z in the domain of $\left|z - (-6-4i)\right| > 10$. (5 marks)

Question three

(a) **Find the centre and the radius of convergence of the following power series :**

$$\sum_{n=0}^{\infty} \frac{(4n)!}{5^n (n!)^4} (z - 6i - 5)^n$$ **(5 marks)**

(b) **Express** $\dfrac{\sin\left(\dfrac{z^2}{3}\right)}{z(z-2)}$ **into its Maclaurin series (i.e., Taylor series with centre at** $z = 0$ **) and find its radius of convergence .** **(5 marks)**

(c) **Given the following definite integral :**

$$\int_0^{2\pi} \frac{\cos(2\theta)}{3 + \cos(\theta)} d\theta$$

(i) **use** int **command to find its value ,** **(4 marks)**

(ii) **convert it to a complex contour integral , evaluate the value of this contour integral . Compare it with that obtained in (i) .** **(11 marks)**

Question four

(a) **Given the following two improper integrals :**

$$\int_{-\infty}^{\infty} \frac{\cos(x)}{x^2 + x + 1} dx \quad \text{and} \quad \int_{-\infty}^{\infty} \frac{\sin(x)}{x^2 + x + 1} dx$$

(i) **convert them into the real and imaginary part of a complex contour integral respectively . Justify your choice of the contour .** **(4 marks)**

(ii) **evaluate the values of the given integrals by the method of residue integration.** **(7 marks)**

(b) **Find the Cauchy principal value of the following integral :**

$$\int_{-\infty}^{\infty} \frac{dx}{x^4 - 81}$$ **(7 marks)**

(c) **Evaluate by the method of residue integration the following improper integral :**

$$\int_{-\infty}^{\infty} \frac{x^2 + 5}{x^4 + x^3 + 2x^2 - x + 3} dx \quad .$$ **(7 marks)**

Question five

(a) **(i)** **Find the linear fractional transformation** $w = \dfrac{az+b}{cz+d}$ **that maps the three points** $\{ i \ , \ 0 \ , \ 1 \}$ **in** $z-plane$ **onto the three points** $\{ 1-i \ , \ -2 \ , \ 3 \}$ **in** $w-plane$ **respectively ,i.e., find the values of** $a \ , \ b \ , \ c \ and \ d$ **.** **(8 marks)**

(ii) **use** $conformal$ **command to plot the mapped image of the rectangular region (** $-20 \le x \le 20$ **and** $3 \le y \le 3.001$ **) of** $z-plane$ **onto the** $w-plane$ **,** **(4 marks)**

(iii) **find the fixed points of the given mapping .** **(3 marks)**

(b) **Find the potential** ϕ **between two infinite coaxial cylinders of radii** $r_1 = 5 \ \ cm$ **and** $r_2 = 10 \ \ cm$ **kept at constant potentials** $U_1 = 40 \ \ volts$ **and** $U_2 = 80 \ \ volts$ **respectively . Also find the potential at** $r = 8 \ \ cm$ **surface .** **(10 marks)**