UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRONIC ENGINEERING

MAIN EXAMINATION 2007/2008

TITLE OF PAPER : NUMERICAL ANALYSIS

COURSE NUMBER : E472

TIME ALLOWED : THREE HOURS

INSTRUCTIONS : ANSWER ANY FOUR OUT OF FIVE QUESTIONS. EACH QUESTION CARRIES 25 MARKS.

MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.

STUDENTS ARE PERMITTED TO USE MAPLE TO ANSWER THE QUESTIONS.

THIS PAPER HAS SIX PAGES, INCLUDING THIS PAGE.

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E472 Numerical Analysis

Question one

Given a polynomial function of x as

$f(x) = x^4 - 9x^3 - 4x^2 - 7x + 250$

(a) plot the given f(x) for $x = 0$ *to* 10 . Use *fsolve* command to find its root or roots in this interval. **(4 marks)**

(b) Transform $f(x) = 0$ into the form $x = g(x)$. Compute a solution of $f(x) = 0$ by fixed-point iteration method, starting from $x_0 = 2$ and doing 4 iterations. Compute the percentage difference of the root found here with the corresponding one obtained in (a). **(7 marks)**

(c) Compute a solution of $f(x) = 0$ by Newton's method, starting from $x_0 = 2$ and doing 4 iterations. Compute the percentage difference of the root found here with the corresponding one obtained in (a). **(6 marks)**

(d) Compute a solution of $f(x) = 0$ by Secant method, starting from $x_0 = 2$ and $x_1 = 2.1$ and doing 4 iterations. Compute the percentage difference of the root found here with the corresponding one obtained in (a). **(8 marks)**

Question two

(a) Given a set of data as $f_0 = f(x_0) = f(-1) = 6.5$, $f_1 = f(x_1) = f(0) = 11.3$,

$f_2 = f(x_2) = f(1) = 8.9$, $f_3 = f(x_3) = f(2) = 2.4$ and $f_4 = f(x_4) = f(3) = 6.1$,

(i) use the Newton's divided difference interpolation to find its interpolated $p_4(x)$,

(7 marks)

(ii) If using a polynomial of power 2 , i.e., $f(x) = k_0 + k_1\, x + k_2\, x^2$, to interpolate the given data , determine the values of k_0 , k_1 *and* k_2 by the method of least squares. **(7 marks)**

(b) Given the following system of linear equations as

$$\begin{cases} 2\,x + 9\,y - 3\,z = -\,39 \\ x - 3\,y + 8\,z = 56 \\ 12\,x + 2\,y - 5\,z = 78 \end{cases}$$

(i) use *linsolve* command to find the solutions of x , y *and* z, **(3 marks)**

(ii) apply the Gauss-Seidel iteration (4 steps) to the given system, starting from $x_1 = 1$, $y_1 = 1$ *and* $z_1 = 1$. Compute the iterated solutions of the system. Compare these values with the solutions obtained in (b) (i) and compute their respective percentage differences. **(8 marks)**

Question three

(a) Given the following definite integral $\int_0^{10} \sqrt{x}\,\sin^2\left(\frac{x}{4}\right) dx$,

(i) use int command to find its value, **(2 marks)**

(ii) divide the integration range into **eight** equal intervals and compute the value of the given integral by the trapezoidal rule. Compare it with that obtained in (a) (i) to find their percentage difference. **(6 marks)**

(iii) divide the integration range into **eight** equal intervals and compute the value of the given integral by Simpson's rule. Compare it with that obtained in (a) (i) to find their percentage difference. **(6 marks)**

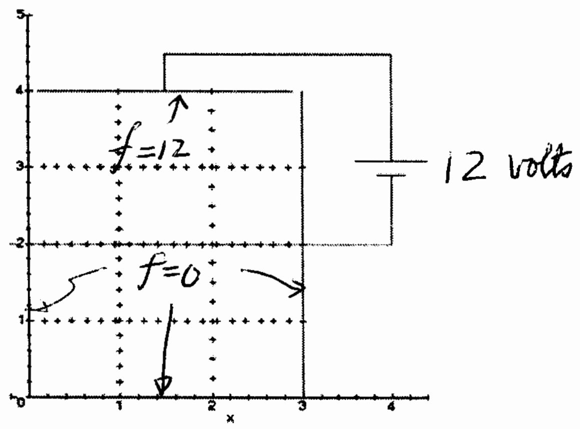
(b) Given the following differential equation $\frac{d\,y(x)}{d\,x} + 5\,y(x) = 7\sin(2x)$ and initial condition $y(0) = 3$,

(i) use *dsolve* command to find its specific solution of $y(x)$. Evaluate the value of $y(2)$. **(3 marks)**

(ii) use Runge-Kutta method by starting with $x = 0$ *and* $h = 0.2$, do 10 steps to find the approximate value of $y(2)$. Compare it with that obtained in (b) (i) to find their percentage difference. **(8 marks)**

Question four

An infinite long, rectangular U shaped conducting channel is insulated at the corners from a conducting plate forming the fourth side with interior dimensions as shown below :



The Dirichlet boundary conditions are given as $f(0,y)=0$, $f(3,y)=0$, $f(x,0)=0$ and $f(x,4)=12$ *volts* .

(a) Use the discrete Laplace equations , i.e.,

$$f(i,j)=\frac{f(i-1,j)+f(i+1,j)+f(i\,,j-1)+f(i\,\,,j+1)}{4} \quad where \quad \begin{array}{c} i=1\,,2 \\ j=1\,,2\,,3 \end{array} ,$$

to find the approximated values of f(1,1) , f(1,2) , f(1,3) , f(2,1) , f(2,2) and f(2,3) .

(10 marks)

(b) Assign the values of f(1,1) , f(1,2) , f(1,3) , f(2,1) , f(2,2) and f(2,3) all as 1. Use the renumeration scheme and do 5 rounds of renumeration to find their approximate values. Compare the value of f(1,1) after 5 rounds of renumeration to that obtained in (a) to find their percentage difference. **(15 marks)**

Question five

(a) Given the following function of x and y as :

$f(x,y) = 3\,x^2 - 2\,x\,y + y^2 - 6\,x + 9$,

(i) find the maximum value of f and the position of (x , y) that the maximum happens by solving $\dfrac{\partial f}{\partial x} = 0 \;\; and \;\; \dfrac{\partial f}{\partial x} = 0$, **(3 marks)**

(ii) use the method of steepest descent , starting with the points $x_0 = 1 \;\; and \;\; y_0 = 1$, do 5 steps to find the approximate maximum value of f and its approximate (x , y) position . Compare these values with those obtained in (a) (i) to find their respective percentage differences. **(10 marks)**

(b) Given the following function of x and y as :

$f(x,y) = 30\,x + 50\,y$ where both x and y are positive variables and are subjected to the following constrains :

$-\,x + y \le 8 \;\; and \;\; 3\,x + y \le 15$,

use the Simplex method to find the localized maximum value of f and the position of (x , y) such that this localized maximum occurs. **(12 marks)**