UNIVERSITY OF SWAZILAND
FACULTY OF SCIENCE
DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

MAIN EXAMINATION 2009

TITTLE OF PAPER: **SIGNALS 1**

COURSE CODE: **E342**

TIME ALLOWED: **THREE** HOURS

INSTRUCTIONS:

1. Answer **any four** of the following **five** questions.
2. Each question carries 25 marks
3. Marks for different sections are shown in the right hand margin

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QUESTION 1

A) Define the following terms:

i) periodic signal, **(2marks)**

ii) discrete-time signal. **(2marks)**

B) For the following signals a) to e) below,

a) $x(t) = 4\cos(5\pi t)$

b) $x(t) = 4\cos(5\pi t - \pi/4)$

c) $x(t) = 4u(t) + 2\sin(3t)$

d) $x[n] = 4\cos(\pi n)$

e) $x[n] = 4\cos(\pi n - 2)$

i. State whether the signal is periodic (if periodic, give period) **(5marks)**

ii. Sketch the signals. (Scale your time axis so that a sufficient amount of the signal is being plotted). **(10marks)**

C) Give an expression for the signals shown in **Figure 1a** and Figure **1b**. **(6 marks)**

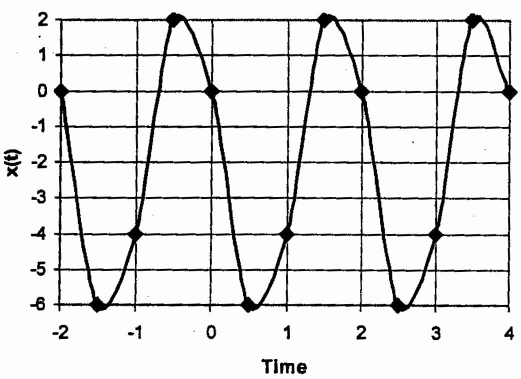


Figure 1a

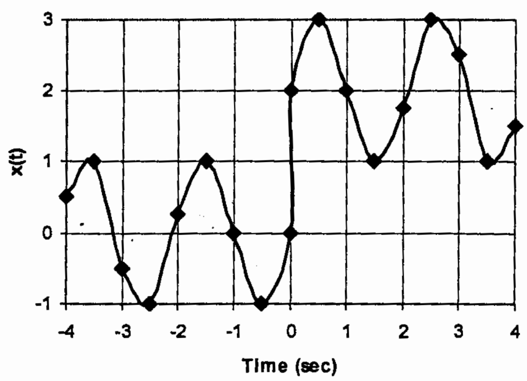


Figure 1b

Figure 1

QUESTION 2

A) Define the terms:

i) signal sampling and **(2 marks)**

ii) sampling period. **(2 marks)**

B) (i) State the sampling theorem and explain the term aliasing error. **(4 marks)**

(ii) Given two signals $a(t) = \cos\omega_0 t$ and $b(t) = \cos(\omega_0 + \omega_s)t$, where $\omega_s = 2\pi f_s$ is the sampling frequency in rad/s. Show that signals $a(t)$ and $b(t)$ are aliased. **(5 marks)**

iii) A Compact Disc (CD) system has a sample rate of $44\,kHz$. What is the highest frequency that can be sampled without aliasing? **(2 marks)**

iv) Samples are to be taken from a record of a continuous-time signal of duration $100\ ms$. The signal contains sinusoidal components with frequencies up to $250\,Hz$. Determine the minimum number of samples that would be sufficient to give a complete representation of the signal. **(4 marks)**

C) Consider a rectangular pulse signal of height A and duration T centered at a point in time $t = t_0 > T$. Sketch the signal waveform in the time domain and obtain an analytic representation in terms of the $rect(\ )$ function and verify that it is correct. **(6 marks)**

QUESTION 3

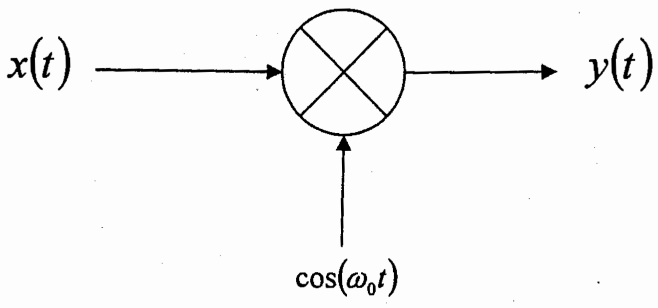
a) Determine whether the following signal processing operations i) to iv) are linear or non linear.

i) $y[n] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]$ **(2 marks)**

ii) $y[n] = \sqrt{x[n]}$ **(2 marks)**

iii) $y(t) = 2x^2(t)$ **(2 marks)**

iv) A system that performs modulation of a carrier signal $\cos(\omega_0 t)$ with an input signal $x(t)$ to produce an output $y(t)$ as shown in **figure 2.**



(4 marks)

Figure 2

b) (i) Explain the term "Time-Invariant" system. **(2 marks)**

(ii) Illustrate how you would test for a system's time-invariance. **(3 marks)**

iii) Explain the term "Linear system" and illustrate how you would test for a system's linearity. . **(4 marks)**

c) Distinguish between power and energy signals and give an example of each. **(6 marks)**

QUESTION 4

A) (i) Define the unit-sample sequence $\delta[n]$ and show how it can be used to express any sequence as the sum of scaled and delayed unit-sample sequences. **(4 marks)**

(ii) Express the sequence given by $x[n]=[1,1,1,1,0,0,0,.....]$ in terms of the unit-sample sequence **(2 marks)**

iii) By using a property of the delta function $\delta(t)$, evaluate the integral $\int_{-\infty}^{\infty} f_1(t)\times f_2(t)dt$,

where $f_1(t)=2\sin(200\pi t)$ and $f_2(t)=\delta(t-0.25\times10^{-3})$. **(2 marks)**

B) (i) Define the Fourier transform of a signal. **(2 marks)**

(ii) The rectangular pulse $x(t)$ shown in **figure 3** is of height A, width τ and symmetrical about the time origin can be expressed as $x(t)=\begin{cases} A & for\ -\tau/2 \le t \le \tau/2 \\ 0 & elsewhere \end{cases}$

Find

a) the spectrum of $x(t)$ **(4 marks)**

b) sketch the spectrum of $x(t)$ **(2 marks)**

iii) An exponentially-decaying sinusoidal voltage has the form $v(t)=Ae^{-\alpha t}\sin\omega_0 t$ for $t\ge0$.

Find the spectrum of this waveform. **(5 marks)**

iv) Use the principle of superposition to find the spectrum of the pulse in **figure 4**. **(4 marks)**

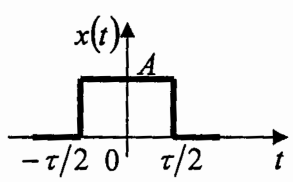


Figure 3

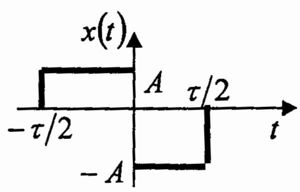


Figure 4

QUESTION 5

A) For the following signal in **figure 5**:

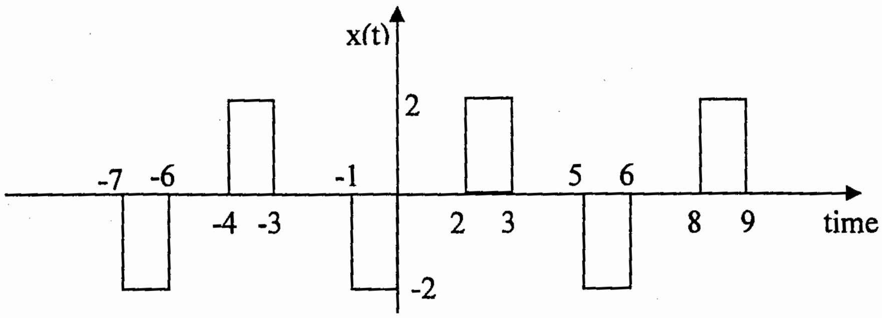


Figure 5

i) Find the Fourier series **(8 marks)**

ii) Plot the spectra versus frequency, $\omega = n\omega_0$. **(4 marks)**

B) A voltage signal is modeled as the sinusoid $v(t) = 5\cos(3t + 0.5)$.

i) Express the signal in terms of exponential frequency components. **(5 marks)**

ii) Sketch the frequency domain representation of the signal. **(2 marks)**

C) Consider the circuit in figure 6.

i) Determine the ratio $\frac{v_{out}}{v_{in}} = G(j\omega)$ **(3 marks)**

ii) Determine and sketch the amplitude and phase characteristics of the circuit in **figure 6**. **(3 marks)**

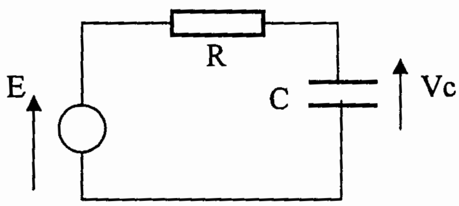


Figure 6

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MAIN EXAMINATION MAY 2009

TITLE OF PAPER: **LINEAR SYSTEMS**

COURSE CODE: **E352**

TIME ALLOWED: **THREE (3)** HOURS

INSTRUCTIONS:

1. Answer question **one** and any other **three** questions.
2. Question one carries 40 marks.
3. Questions 2, 3, 4, and 5 carry 20 marks each.
4. Marks for different sections are shown in the right-hand margin

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Question 1

(A) The relationship between the input x(t) and the output y(t) for a nonlinear system is given by the equation $\mathbf{y(x) = x^2 + x\sin(x)}$. Obtain an approximate linear equation representing this system at an operating point $x_0 = 2$.

(13 marks)

(B) An input $\mathbf{r(t) = 1}$ $\boldsymbol{for\ t \geq 0}$ is applied to a black box with a transfer function G(s) when the initial conditions are zero. The resulting output response is $\mathbf{y(t) = t + 0.5t^2 + (1/3)\sin(3t)}$ Obtain the transfer function G(s). (6 marks)

(C) (i) Why is convolution important in the analysis of linear time-invariant systems?

(2marks)

(ii) In case of linear time-invariant state-space models what is linear transformation of states and why is it useful? (3 marks)

(D) The transfer function of a linear system is $\dfrac{Y(s)}{R(s)} = \dfrac{10(s+3.74)}{s^2+6s+34}$.

If r(t) is a unit step input,

(i)determine the response y(t) (10 marks)

(ii) determine the rise time, and (3 marks)

(iii) determine the settling time for 5% tolerance.. (3 marks)

Question 2

(A) A linear system is represented by the Equation

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.5 \end{bmatrix} u$$

where u is a unit step.

(i) Find the matrix $\Phi(t)$ (8 marks)

(ii) For the initial conditions $x_1(0) = 1$ and $x_2(0) = 2$ find $\boldsymbol{x}(t)$. (12 marks)

Question 3

A continuous time-invariant linear system is presented by the following model

$$\dot{x} = \begin{bmatrix} 1 & 2 & 0 \\ -7 & -2 & -36 \\ -1 & 0 & -7 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0.4 & 5 \end{bmatrix} x$$

Obtain the diagonal form realization of this system.. [20 marks]

Question 4

Use **Mason's gain rule** to find the transfer function of a fuel-injection engine system model whose block diagram is shown in Figure 4. (20 marks)

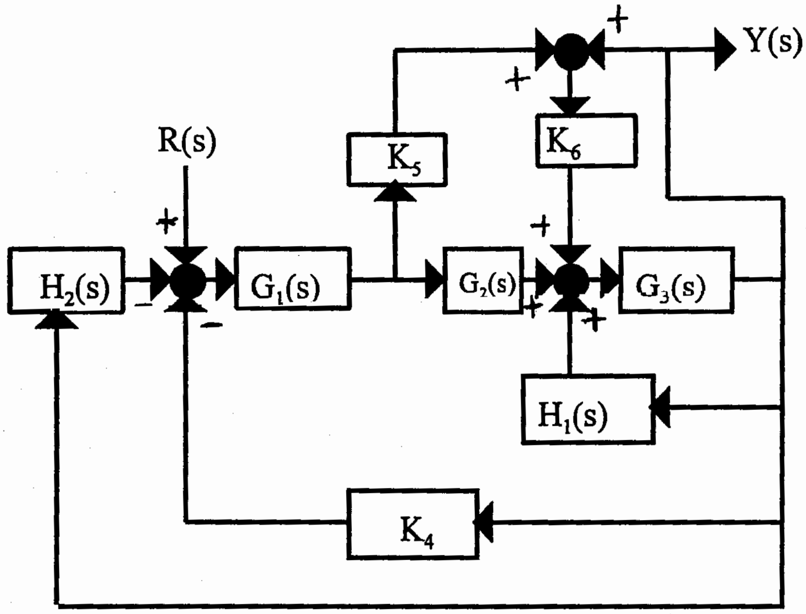

Figure 4

Question 5

For an equivalent circuit of a two-transistor series voltage feedback amplifier shown in Figure 5.

(i) Use **block diagram reduction method** to determine the voltage gain $\frac{V_o(s)}{V_{in}(s)}$

(10 marks)

(ii) Determine the transfer function $\frac{V_{in}(s)}{I_{b1}(s)}$ representing the input impedance.

(10 marks)

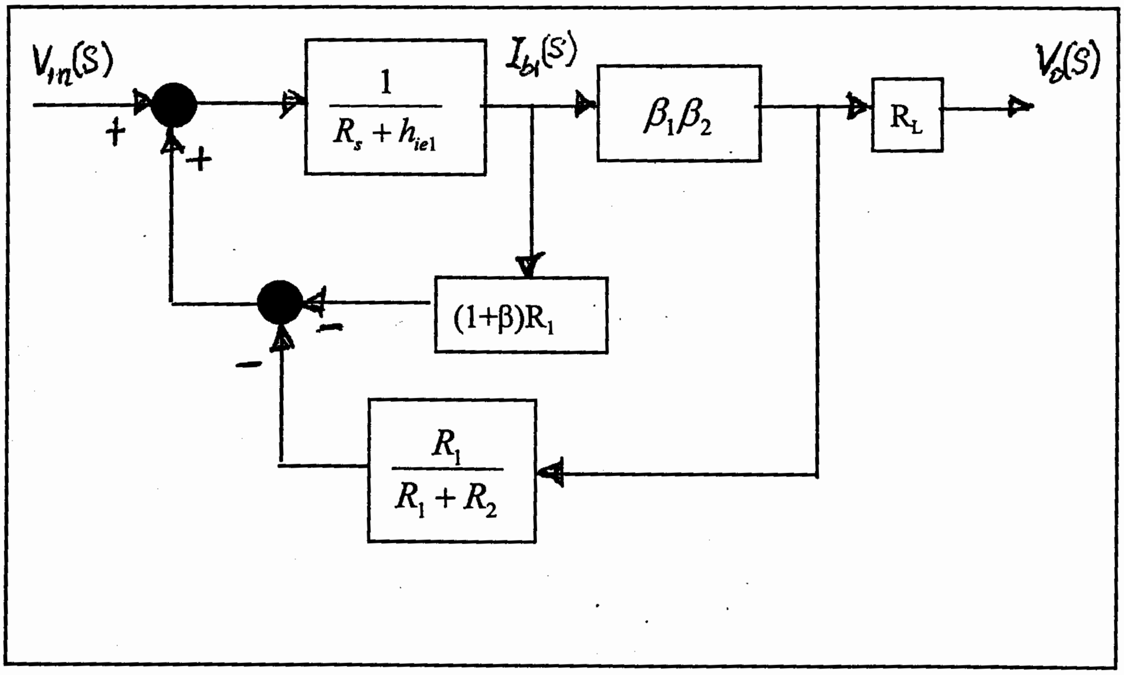

Figure 5