UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRONIC ENGINEERING

MAIN EXAMINATION 2008/2009

TITLE OF PAPER	:	**ORDINARY DIFFERENTIAL EQUATIONS, PROBABILITY AND STATISTICS**
COURSE NUMBER	:	**E371**
TIME ALLOWED	:	**THREE HOURS**
INSTRUCTIONS	:	**ANSWER ANY FOUR OUT OF FIVE QUESTIONS. EACH QUESTION CARRIES 25 MARKS.** **MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.** **STUDENTS ARE PERMITTED TO USE MAPLE TO ANSWER THE QUESTIONS.**

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E371 Ordinary Differential Equations, Probability and Statistics

Question one

A non-homogeneous ordinary differential equation is given as

$$2\frac{d^2 y(t)}{dt^2} + 3\frac{d\,y(t)}{dt} + 7\,y(t) = 4\,e^{-2t} - 3\cos(5t)$$

(a) Find its particular solution $y_p(t)$. Plot $y_p(t)$ for $t = 0$ *to* 10. **(9 marks)**

(b) Find the general solution $y_h(t)$ for the homogeneous part of the given differential equation, and then write down the general solution $y_g(t)$ for the above given non-homogeneous differential equation. **(4 marks)**

(c) If the initial conditions are given as $y(0) = -6$ *and* $\left.\frac{d\,y(t)}{dt}\right|_{t=0} = 3$,

find the specific solution of $y(t)$ and plot it for $t = 0$ *to* 10 . Compare this diagram with the one in (a) and make brief comment. **(12 marks)**

Question two

(a) If the inverse laplace transform of $F(s)$ is $5\sin(4t)$,

(i) find $F(s)$, **(2 marks)**

(ii) find the inverse laplace transform of $e^{-3s} F(s)$ by utilizing t – shift theorem and plot it for $t = 0$ *to* 6 . **(5 marks)**

(b) Given the following differential equation as

$$\frac{d^2 y(t)}{dt^2} + \frac{d\,y(t)}{dt} + 3\,y(t) = f(t)$$

where
$$f(t) = \begin{cases} 0 & \text{if} \quad t \le 0 \\ \dfrac{t^2}{9} & \text{if} \quad 0 \le t \le 3 \\ -\dfrac{t}{6} + \dfrac{3}{2} & \text{if} \quad 3 \le t \le 9 \\ 0 & \text{if} \quad t \ge 9 \end{cases}$$

(i) plot the given $f(t)$ for $t = 0$ *to* 10 , **(3 marks)**

(ii) find the laplace transform of the above given $f(t)$, **(3 marks)**

(iii) if given the initial conditions as $y(0) = -5$ *and* $\left.\frac{d\,y(t)}{dt}\right|_{t=0} = 3$, find the laplace transform of $y(t)$, **(7 marks)**

(iv) find the specific solution of $y(t)$ through inverse laplace transform of your answer in (b) (iii). Plot this $y(t)$ for $t = 0$ *to* 10 . **(5 marks)**

Question three

Given the following differential equation as

$$5\frac{d^2 y(x)}{dx^2} + 4\frac{d\,y(t)}{dt} + 8\,y(t) = 0$$

set $y(x) = \sum_{n=0}^{\infty} a_n\, x^{n+s}$ *and* $a_0 \neq 0$, utilize the power series method and

(a) write down the indicial equations. Set $a_0 = 1$ and solve for the values of s and a_1 , **(7 marks)**

(b) write down the recurrence relation and use them to find the values of a_n ($n = 2$ *to* 10) for each value of s found in (a). Write down two independent solutions in polynomial form truncated up to a_{10} term.

(8 marks)

(c) (i) Write the general solution for the above given differential equation.

(2 marks)

(ii) If the initial conditions are given as $y(0) = -1$ *and* $\left.\frac{d\,y(x)}{dx}\right|_{x=0} = 2$, find the specific solution and plot it for $x = 0$ *to* 5 . **(8 marks)**

Question four

(a) Given a discrete probability function $f(1)=\frac{1}{12}$, $f(2)=\frac{1}{6}$, $f(3)=\frac{1}{2}$, $f(4)=\frac{1}{6}$ and $f(5)=\frac{1}{12}$, find its probability distribution function G(x), i.e., find the values of G(1), G(2), G(3), G(4) and G(5). Plot a bar chart of f(x) for $x=0$ *to* 5 . **(6 marks)**

(b) (i) Use the random number generator to generate an ensemble of 20 data of x with its values ranging from 26 to 65, then find its mean value and standard deviation. **(6 marks)**

(ii) using the interval of 10 starting with 25.5, i.e., (25.5 to 35.5), (35.5 to 45.5), ……., (65.5 to 75.5), plot its histogram. **(5 marks)**

(c) Ten identical coins are tossed simultaneously and each coin has its probability of "head up" in a toss as 0.495,

(i) find the probability of precisely 3 heads up, **(4 marks)**

(ii) find the probability of at least 3 heads up. **(4 marks)**

Question five

(a) If the defect rate for a skew production is 1 out of 95 and one picks up a handful of 300 skews, use Poisson distribution to

(i) find the probability of exactly 1 defected skew being picked up.

(4 marks)

(ii) find the probability of no more than 2 defected skews being picked up.

(6 marks)

(b) For a six inch nail production factory, assuming its produced nail lengths follow a normal distribution with the mean value of 6 ,

(i) if the standard deviation of the factory products is 0.02 and the confidence level is 0.98 , then what would be the corresponding confidence range?

(7 marks)

(ii) if the required confidence level and confidence range by the customer are 0.98 and $5.98 \leq \textit{nail length} \leq 6.02$, then what should be the maximum standard deviation of the nail production that can meet such demands.

(8 marks)