UNIVERSITY OF SWAZILAND

MAIN EXAMINATION DECEMBER, 2008

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRONIC ENGINEERING

TITLE OF PAPER:	CONTROL SYSTEMS
COURSE CODE:	E430
TIME ALLOWED:	THREE HOURS

INSTRUCTIONS:

1. **Answer question ONE and any other THREE questions**
2. **Question one carries 40 marks.**
3. **Questions 2, 3, 4, and 5 carry 20 marks each.**
4. **Mark for different sections are shown in the right-hand margin.**
5. **Linear graph paper and Linear-Log paper are provided**

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR

THIS PAPER CONTAINS EIGHT (8) PAGES INCLUDING THIS PAGE.

Partial Table of z- and s-Transforms

	f(t)	F(s)	F(z)	f(kt)
	$u(t)$	$\frac{1}{s}$	$\frac{z}{z-1}$	$u(kT)$
2.	t	$\frac{1}{s^2}$	$\frac{Tz}{(z-1)^2}$	kT
3.	t^n	$\frac{n!}{s^{n+1}}$	$\lim_{a\to 0}(-1)^n \frac{d^n}{da^n}\left[\frac{z}{z-e^{-aT}}\right]$	$(kT)^n$
4.	e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$	e^{-akT}
5.	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$	$(-1)^n \frac{d^n}{da^n}\left[\frac{z}{z-e^{-aT}}\right]$	$(kT)^n e^{-akT}$
6.	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$	$\frac{z \sin \omega T}{z^2 - 2z\cos\omega T + 1}$	$\sin \omega kT$
7.	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$	$\frac{z(z-\cos\omega T)}{z^2 - 2z\cos\omega T + 1}$	$\cos \omega kT$
8.	$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$	$\frac{ze^{-aT}\sin\omega T}{z^2 - 2ze^{-aT}\cos\omega T + e^{-2aT}}$	$e^{-akT}\sin \omega kT$
9.	$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$	$\frac{z^2 - ze^{-aT}\cos\omega T}{z^2 - 2ze^{-aT}\cos\omega T + e^{-2aT}}$	$e^{-akT}\cos \omega kT$
10.			$\frac{z}{z+a}$	$a^k \cos k\pi$

z-Transform Theorems

	Theorem	Name
1.	$z\{af(t)\} = aF(z)$	Linearity theorem
2.	$z\{f_1(t) + f_2(t)\} = F_1(z) + F_2(z)$	Linearity theorem
3.	$z\{e^{-at}f(t)\} = F(e^{aT}z)$	Complex differentiation
4.	$z\{f(t-nT)\} = z^{-n}F(z)$	Real translation
5.	$z\{tf(t)\} = -Tz\frac{dF(z)}{dz}$	Complex differentiation
6.	$f(0) \doteq \lim_{z\to\infty} F(z)$	Initial value theorem
7.	$f(\infty) = \lim_{z\to 1}(1-z^{-1})F(z)$	Final value theorem

Question 1

(a) Economic inflation is a positive feedback system. In this system, the input signal called ***initial wages*** is added to ***wage increase*** to *produce* ***actual wages.*** The signal ***actual wages*** is used as an input to a process called ***industry*** of which the output is ***prices***. ***Prices*** multiplied by a gain K_1 is equal to ***cost of living***. The ***cost of living*** is manipulated by ***Automatic cost of living increase*** from which ***wage increase*** is obtained. Draw a closed-loop control system block diagram with all signals, gains, and transmittances labelled. [8 marks]

(b) A block diagram of a rate loop for a missile system is shown in Figure Q1B. If the desired rate is a unit step, then write a MATLAB script for plotting the actual rate $y(t)$ for $0<t<3$ seconds and at 0.01 seconds time intervals.

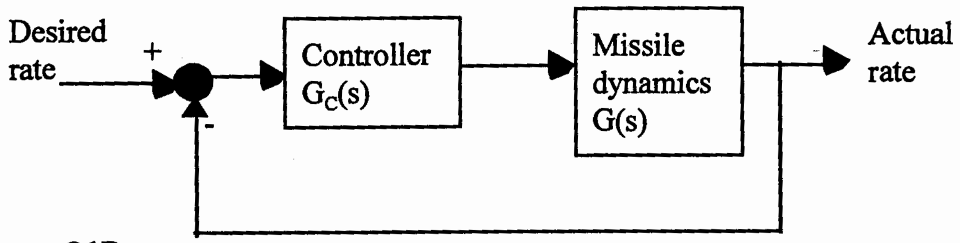


Figure Q1B

$$G_C(s) = 0.1 + \frac{5}{s} \; ; \qquad G(s) = \frac{100(s+1)}{s^2 + 2s + 100} .$$

[13 marks]

(c) Given that a continuous time derivative can be approximated in discrete time by using a backward difference rule as follows $\left.\frac{dx(t)}{dt}\right|_{t=KT} = \frac{1}{T}\{x(KT) - x[(K-1)T]\}$

and the integration of x(t) by the forward- rectangular integration at $t = KT$ is

$r(KT) = r[(K-1)T] + Tx(KT)$ where $r(KT)$ is the output of the integrator at $t = TK$,

determine the z-transform of a **PID controller** with an s-domain transfer function

$$G_C(s) = 2 + \frac{0.1}{s} + 0.5s .$$

[7 marks]

Question 1 (continued)

(d) Determine the necessary gain K_3 in metres required to maintain a steady state tracking error equal to 1 centimetre between the desired and actual position for the gas-jet propulsion system shown in Figure Q1D when the input is a unit ramp in metres.

[12 marks]

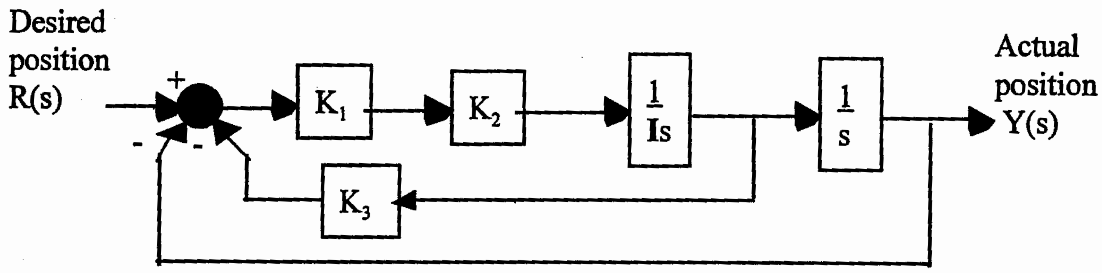


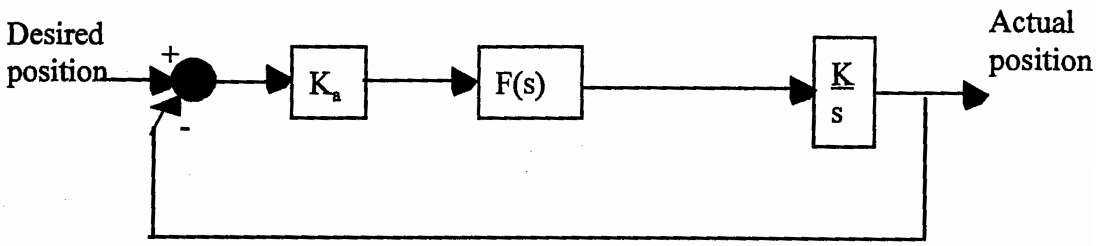
Figure Q1D

Where **I** is the inertia constant

Question 2

A linear model of a phase-lock loop is shown in Figure Q2. The limit for the stability of this system is set by the gain K_aK which is equal to the velocity constant K_v.
(i) Determine the range of K_v for which the system remains stable.
(ii) Determine the value of K_v and the location of the roots when the steady state error is 1^0 and ramp input signal of 100 rad/s is applied to the system.

[20 marks]



$$F(s) = \frac{10(s+10)}{(s+1)(s+100)}$$

Figure Q2

Question 3

(a) A two dimensional control system is shown in Figure Q3A, in which a set of state variables is defined. Obtain the state differential equations and present them in a matrix form.

[10 marks]

(b) For the magnetic disk drive system shown in Figure Q3B, determine sensitivity of this system due a small change in time τ and then the equation for the corresponding change for the closed -loop transfer function due to the change in τ . [10 marks]

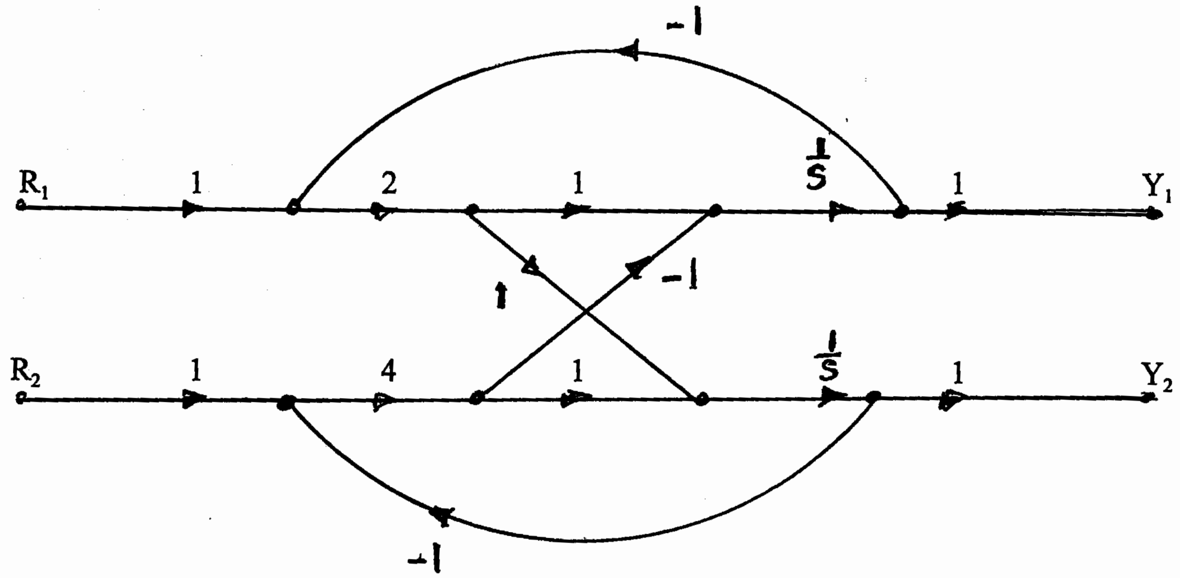


Figure Q3A

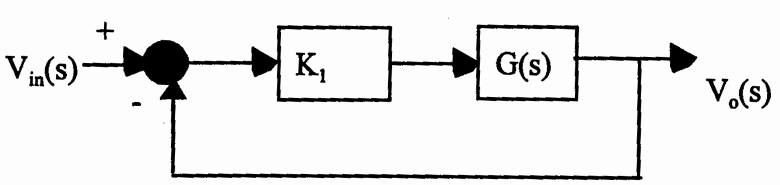


Figure Q3B

$$G(s) = \frac{10}{s(\tau s + 1)}$$

Question 4

A control system for controlling pressure in a chamber has a loop transfer function

$$GH(s) = \frac{30000(2s+1)}{s(s+10)(s+20)(s^2+15s+150)}$$

Draw to scale the Bode plot (Magnitude and Phase plots) of the system. [20 marks]

Question 5

For the open-loop transfer function $GH(s) = \dfrac{k}{(s^2 + 2s + 2)(s+1)}$ of the system shown in Figure Q5, draw to scale the root locus. [20 marks]

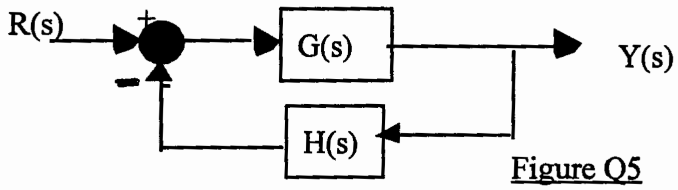
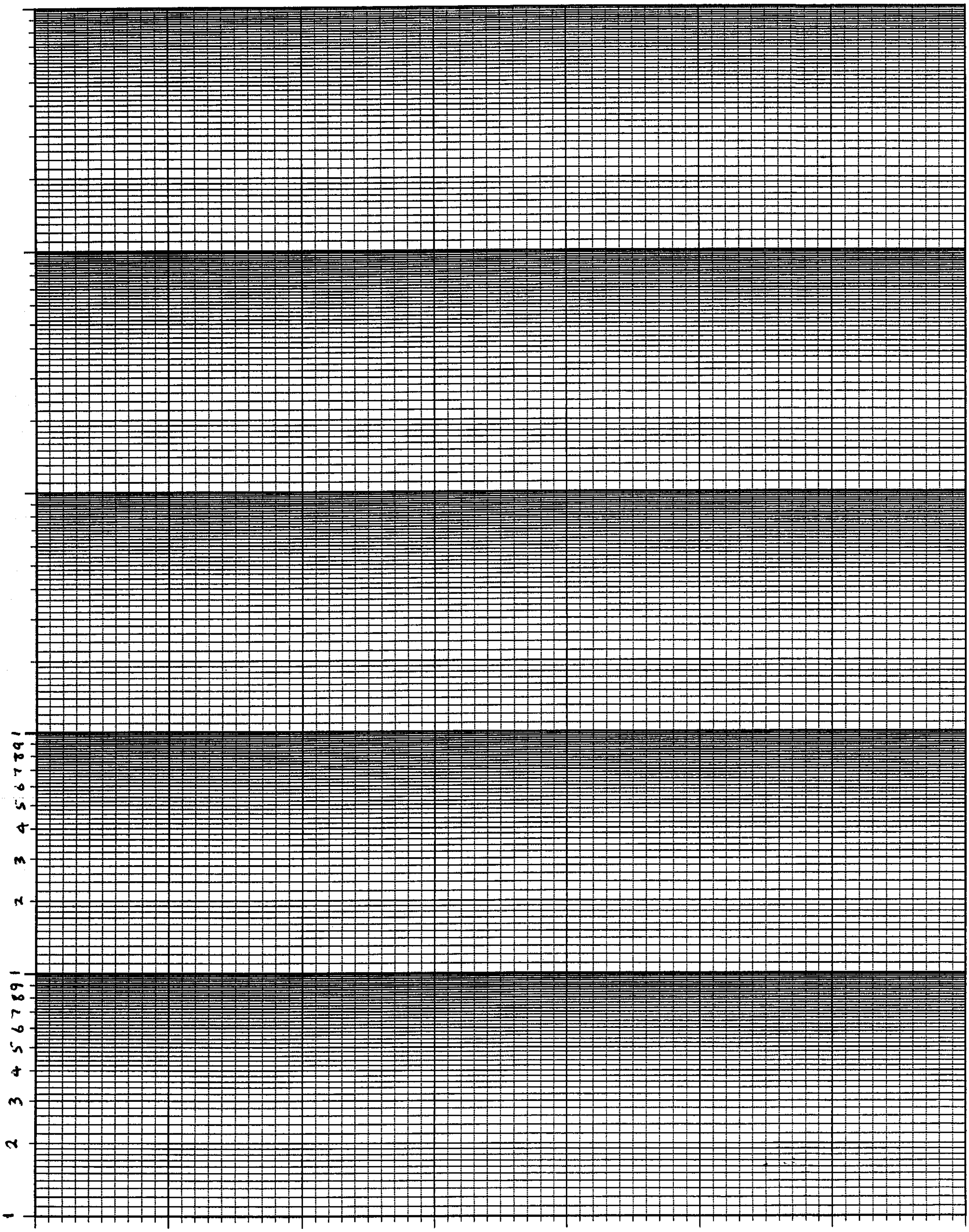


Figure Q5

Five Cycle Semi-Log



Five Cycle Semi-Log