University of Swaziland

Faculty of Science

Department of Electrical and Electronic Engineering

Examinations: May 2009

Title of Paper: **Signals II**

Course Code: **E462**

Duration: **Three hours**

Instructions:
1. Answer any **FOUR QUESTIONS.**
2. Each question carries 25 marks.
3. Useful tables are attached at the end of the question paper

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THIS PAPER CONTAINS EIGHT (6) PAGES INCLUDING THIS PAGE

Question 1

(a) A signal $x(t)$ has the Fourier transform $X(f)$ given by

$$X(f)=\frac{20}{8+j4\pi f}$$

A new signal $y(t)$ is related to $x(t)$ through

$$y(t)=x(2t+2).$$

(i) Find $Y(f)$.

(ii) Derive an expression of $y(t)$ [not as a function of $x(t)$].

[12 marks]

(b) If $V(f)=AT\frac{\sin(2\pi fT)}{2\pi fT}$, find the energy E contained in v(t), where v(t) is the inverse Fourier Transform of V(f).

[5 marks]

(c) Consider the two signal x(t) and y(t) given below:

$$x(t) = \Sigma_{n=0}^{1} 4\delta(t-0.025n) \ .$$

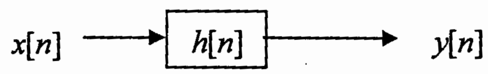
$$y(t) = \Sigma_{n=-1}^{n=1} 4\delta(t-0.025n)$$

Obtain the convolution of x(t) and y(t)

[8 marks]

Question 2

(a) A linear time invariant (LTI) system is shown below.



Let $x[n]=2\delta[n-1]-0.5\delta[n-3]$ and $h[n]=2\delta[n]+\delta[n-1]-3\delta[n-3]$.
Find the output sequence y(n).

[8 marks]

(b) A complex sequence is given by

$$\{h(n)\} = \left\{ -2+j5 \quad 4-j3 \quad \underset{\uparrow}{5+j6} \quad 3+j \quad -7+j2 \right\}$$

Determine

(iii) the conjugate symmetric part,
(iv) the conjugate anti-symmetric part,
(v) the energy of the sequence {h(n)}, and
(vi) the power of the sequence {h(n)}

[17 marks]

Question 3

(a) The signal $x(t) = 2 + 2\cos(60\pi t) + \cos(100\pi t)$ is sampled at a frequency of 90Hz. The sampled signal is then passed through an ideal unity-gain low-pass filter which has a bandwidth of 55Hz. Assume natural sampling with sampling pulses of $\tau = 4.5$ ms.

(i) Sketch the power spectral density of the signal at the output of the low-pass filter clearly showing the frequencies power values.
(ii) Obtain the power of the signal at the output of the low-pass filter.

[13 marks]

(b) A voltage signal $x(t) = 8 + 12\cos(60\pi t)$ is applied to the system shown in **Fig.Q3(b)**. If $n(t)$ is additive noise with power spectral density

$$G_n(f) = \begin{cases} 0.06 & 0 \le |f| \le 3000 \\ 0 & \textit{otherwise} \end{cases},$$

obtain the signal to noise ratio (in dBs) at point A on **Fig.Q3(b)**. The ideal low pass filter has a unity passband gain and bandwidth of 80Hz.

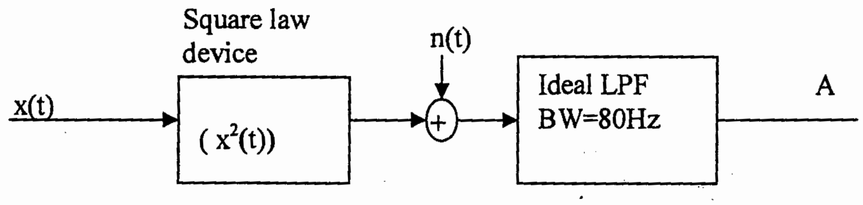


Fig.Q3(b).

[12 marks]

Question 4

(a) An M-point moving average system is given by $y(n)=\frac{1}{M}\sum_{k=0}^{M-1}x(n-k)$.

Show whether the M-point moving average system is
(i) linear, (ii) time invariant, (iii) causal or (iv) bounded-in-bounded-out (BIBO) stable.

[21 marks]

(b) A length-7 sequence is given by $x(n)=\begin{pmatrix} 3 & -2 & 0 & 1 & 4 & 5 & 2 \\ \uparrow & & & & & & \end{pmatrix}$.

Express the sequence as a linear combination of a delayed unit sample sequences.

[4 marks]

Question 5

(a) Two random variables A and B have the probability density functions given in **Fig. Q5(a)**. The variables A and B are correlated with correlation coefficient ρ=0.4. Consider a new random variable Y = A+2B

(i) Obtain the mean of Y
(ii) Obtain the variance of Y

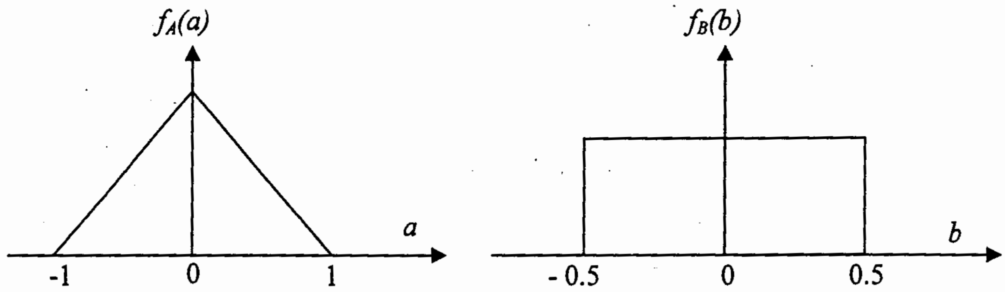


Fig. Q5(a)

[13 marks]

(b) A random variable X is defined by the cumulative distribution function

$$F(x) = \begin{cases} 0 & x<0 \\ 0.5x & 0<x\le 1 \\ 0.25x+0.25 & 1<x\le 3 \\ K & 3\le x \end{cases}$$

(i) Find the value of K
(ii) Find the probability that X lies between 0.5 and 2
(iii) Sketch the probability density function of the variable X.

[12 marks]

Question 6

(a) Consider the random process

$$n(t) = A\cos(2\pi f_o t + \phi)$$

where

- f_o is a constant,
- A is a zero mean Gaussian random variable with variance σ^2,
- ϕ is a random variable with uniform probability density function

$$f(\phi) = \begin{cases} 2/\pi & \ldots\ldots\ldots \quad |\phi| \le \pi/4 \\ 0 & \ldots\ldots\ldots \quad \textit{otherwise} \end{cases}$$

- ϕ is independent of A.

Compute the (i) mean and (ii) autocorrelation of n(t)

[16 marks]

(b) A binary transmission system uses the waveforms $p_o(t)$ and $p_1(t)$ shown in Fig.Q6(b) to transmit "0" and "1" respectively. The transmitted "0"s and "1"s occur independently with equal probability and at a rate of 1000 bits/s.

(i) Obtain the average value of the transmitted waveform.
(ii) Obtain the power spectral density of the transmitted waveform.

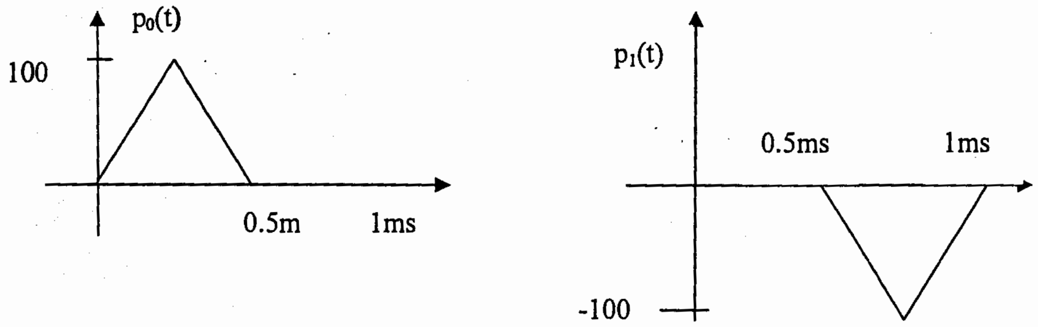
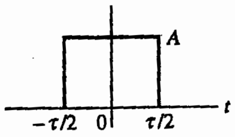
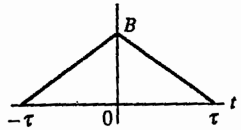
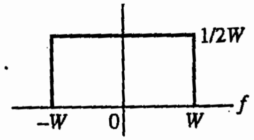


Fig.Q6(b)

[9 marks]

.5 FOURIER TRANSFORM PAIRS

1.	 $\triangleq A\Pi(t/\tau)$	$A\tau \dfrac{\sin \pi f\tau}{\pi f\tau} = A\tau \,\mathrm{sinc}\, \pi f\tau$
2.	 $\triangleq B\Lambda(t/\tau)$	$B\tau \dfrac{\sin^2 \pi f\tau}{(\pi f\tau)^2} = B\tau \,\mathrm{sinc}^2 f\tau$
3.	$e^{-\alpha t}u(t)$	$\dfrac{1}{\alpha + j2\pi f}$
4.	$\exp(-\lvert t\rvert/\tau)$	$\dfrac{2\tau}{1+(2\pi f\tau)^2}$
5.	$\exp[-\pi(t/\tau)^2]$	$\tau \exp[-\pi(f\tau)^2]$
6.	$\dfrac{\sin 2\pi Wt}{2\pi Wt} \triangleq \mathrm{sinc}\, 2Wt$	 $= \dfrac{\Pi(f/2W)}{2W}$
7.	$\exp[j(\omega_c t + \phi)]$	$\exp(j\phi)\delta(f-f_c),\ \omega_c = 2\pi f_c$
8.	$\cos(\omega_c t + \phi)$	$\frac{1}{2}\delta(f-f_c)\exp(j\phi) + \frac{1}{2}\delta(f+f_c)\exp(-j\phi)$
9.	$\delta(t-t_0)$	$\exp(-j2\pi f t_0)$
10.	$\sum_{m=-\infty}^{\infty} \delta(t - mT_s)$	$\frac{1}{T_s}\sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_s}\right)$
11.	$\mathrm{sgn}\, t = \begin{cases} +1, & t>0 \\ -1, & t<0 \end{cases}$	$-\dfrac{j}{\pi f}$
12.	$ut = \begin{cases} 1, & t>0 \\ 0, & t<0 \end{cases}$	$\frac{1}{2}\delta(f) + \dfrac{1}{j2\pi f}$
13.	$\hat{x}(t)$	$-j\,\mathrm{sgn}(f)X(f)$

G.2 TRIGONOMETRIC IDENTITIES

Euler's theorem: $e^{\pm ju} = \cos u \pm j\sin u$

$\cos u = \frac{1}{2}(e^{ju} + e^{-ju})$

$\sin u = (e^{ju} - e^{-ju})/2j$

$\sin^2 u + \cos^2 u = 1$

$\cos^2 u - \sin^2 u = \cos 2u$

$2\sin u \cos u = \sin 2u$

$\cos^2 u = \frac{1}{2}(1 + \cos 2u)$

$\sin^2 u = \frac{1}{2}(1 - \cos 2u)$

$$\cos^{2n} u = \left[\sum_{k=0}^{n-1} 2\binom{2n}{k}\cos 2(n-k)u + \binom{2n}{n}\right] \Big/ 2^{2n}$$

$$\cos^{2n-1} u = \left[\sum_{k=0}^{n-1} \binom{2n-1}{k}\cos(2n-2k-1)u\right] \Big/ 2^{2n-2}$$

$$\sin^{2n} u = \left[\sum_{k=0}^{n-1} (-1)^{n-k} 2\binom{2n}{k}\cos 2(n-k)u + \binom{2n}{n}\right] \Big/ 2^{2n}$$

$$\sin^{2n-1} u = \left[\sum_{k=0}^{n-1} (-1)^{n+k-1}\binom{2n-1}{k}\sin(2n-2k-1)u\right] \Big/ 2^{2n-2}$$

where

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$

$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$

$\sin u \sin v = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$

$\cos u \cos v = \frac{1}{2}[\cos(u-v) + \cos(u+v)]$

$\sin u \cos v = \frac{1}{2}[\sin(u-v) + \sin(u+v)]$