UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRONIC ENGINEERING

MAIN EXAMINATION          2008/2009

TITLE OF PAPER     :     COMPLEX VARIABLES

COURSE NUMBER  :     E471

TIME ALLOWED     :     THREE HOURS

INSTRUCTIONS     :     ANSWER ANY FOUR OUT OF FIVE
                              QUESTIONS. EACH QUESTION
                              CARRIES 25 MARKS.

                              MARKS FOR DIFFERENT SECTIONS
                              ARE SHOWN IN THE RIGHT-HAND
                              MARGIN.

                              STUDENTS ARE PERMITTED TO USE
                              MAPLE TO ANSWER THE
                              QUESTIONS.

THIS PAPER HAS EIGHT PAGES, INCLUDING THIS PAGE.

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GIVEN BY THE INVIGILATOR.

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## E471 Complex Variables

### Question one

(a)    Given   $u(x,y) = a x^3 - 9 x y^2$ ,

  (i)    find the appropriate value of  $a$  such that the given  $u(x,y)$  is a

  harmonic function ,                                                                ( 3 marks )

  (ii)   find its conjugate harmonic function ,  $v(x,y)$ .          ( 5 marks )

(b)    Given   $f(z) = \dfrac{\sinh(2z)}{z-2}$  ,   $z_1 = -2 - 6i$   and   $z_2 = 4 + 6i$  ,

  find the value of   $\displaystyle\int_{z_1,L}^{z_2} f(z)\,dz$

  (i)    if  L  is a straight line from  $z_1$  to  $z_2$  ,              ( 8 marks )

  (ii)   if  L  is a parabolic line path from  $z_1$  to   $z_2$  described by

  $y = x^2 - 10$  . Compare this result with that obtained in (b)(i) and make

  brief comment.                                                            ( 9 marks )

## Question two

(a) Given $f(z) = \dfrac{6i}{z^2 + 2z + 10}$ ,

     (i)     convert the given $f(z)$ into its partial fraction,         ( 3 marks )

     (ii)    given the expansion centre as $z_0 = 1 + 2i$ , find its Laurent series and

            specify the region such that this Laurent series is convergent.

                                                                   ( 9 marks )

(b) Given a definite integral of $\displaystyle\int_0^{2\pi} \dfrac{\sin(2\theta) - \cos(2\theta)}{7 - 4\sin(\theta)}\, d\theta$ ,

     (i)     use int command to find its value,             ( 2 marks )

     (ii)    convert it to a complex contour integral and utilize the residue theorem to

            find its value. Compare this value with that obtained in (b)(i) and make a

            brief comment.                                          ( 11 marks )

## Question three

Convert the following definite integrals into complex contour integrals and utilize the

residue theorem to find

(a)   the value of   $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{x^4 + 3x^3 + 6x^2 + 6x + 4}$   (6 marks)

(b)   the values of   $\displaystyle\int_{-\infty}^{\infty} \frac{\cos(2x)}{x^2 + x + 5}\,dx$   and   $\displaystyle\int_{-\infty}^{\infty} \frac{\sin(2x)}{x^2 + x + 5}\,dx$   (10 marks)

(c)   the principal value of   $\displaystyle\int_{-\infty}^{\infty} \frac{x}{8 - x^3}\,dx$   .   (9 marks)
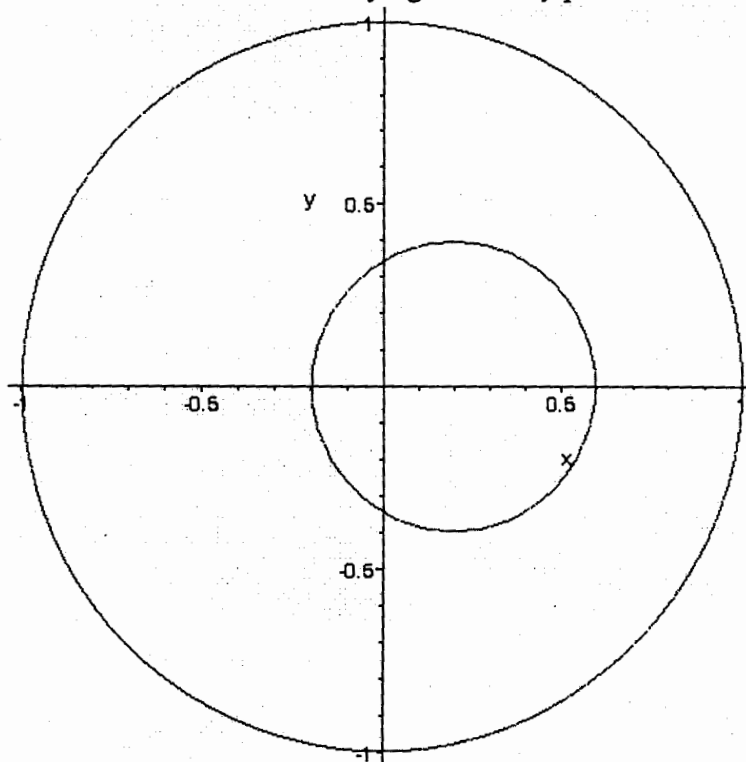
**Question four**

(a)     Find the linear fractional transformation $w = \dfrac{a\,z + b}{c\,z + d}$ such that it maps

$$\left( z_1 = 1 \; to \; w_1 = \frac{1}{3} \right) \;\; , \;\; \left( z_2 = 0 \; to \; w_2 = -\frac{3}{4} \right) \;\; \& \;\; \left( z_3 = 5 \; to \; w_1 = \frac{1}{7} \right)$$

**( 6 marks )**

(b)     Two long hollow metal cylinders eccentrically located one inside the other have their circular cross sections lying on the x-y plane as shown below :



The outer cylinder has a radius of unity and centered at the origin while the inner one has a radius of 0.4 and centered at ( 0.2 , 0 ).

(i)     Find the appropriate value of $b$ in $w = \dfrac{z - b}{b\,z - 1}$ such that it can map these two off-centered cylinders in the z-plane to a pair of coaxial cylinders in the $w$-plane. Find also the radius of the mapped inner cylinder in the $w$-plane.

**( 8 marks )**

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**Question four (continued)**

(ii)     If the electric potential of the outer cylinder is 3 volts and that of the

inner one is zero, find the potential between the cylinders. Plot the equal

potential surfaces of 0 , 1 , 2 and 3 volts in the z-plane and show them

in a single display.                             **( 11 marks )**

## Question five

(a)     Given a linear fractional transformation     $w = \dfrac{z+4}{z-4}$ ,

   (i)      find its inverse transformation ,                              ( 2 marks )

   (ii)     use *conformal* command to plot  both  $|w| = 0.2$  and  $|w| = 5$  circles

            in  $w-$ plane  onto their mapping in the $z-$ plane. Show them in a single

            display.                                                       ( 6 marks )

(b)     Two long parallel transmission lines with the same cross-section radius of

        5 and a separation between their central axis of  20 , with their circular cross

        sections lying on the x-y plane as  shown below :



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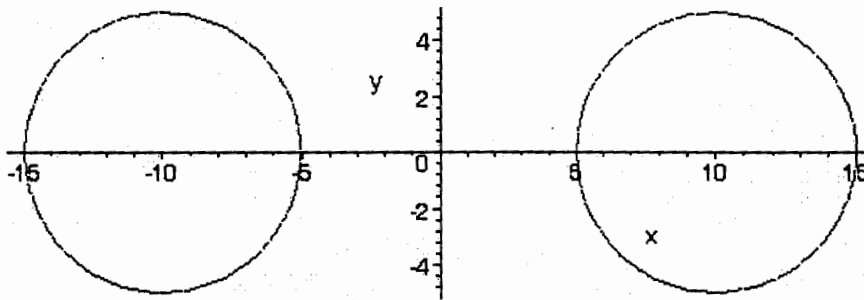
(i)     Find the appropriate value of $c$ given in $w = \dfrac{z+c}{z-c}$ such that it can

map these two transmission line in the z-plane into a pair of coaxial cables

in the $w$-plane. Find also the radius of these coaxial cables.( **7 marks** )

(ii)    If the electric potential of the left line is zero and that of the right one is $5$,

find the potential between the transmission lines. Plot the equal

potential surfaces of $0, 1, 3$ and $5$ volts in the z-plane and show them

in a single display.                                   ( **10 marks** )