UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRONIC ENGINEERING

MAIN EXAMINATION 2008/2009

TITLE OF PAPER	:	NUMERICAL ANALYSIS
COURSE NUMBER	:	E472
TIME ALLOWED	:	THREE HOURS
INSTRUCTIONS	:	ANSWER ANY FOUR OUT OF FIVE QUESTIONS. EACH QUESTION CARRIES 25 MARKS. MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN. STUDENTS ARE PERMITTED TO USE MAPLE TO ANSWER THE QUESTIONS.

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E472 Numerical Analysis

Question one

Given a polynomial function of x as

$$f(x) = 5x^3 - 21x^2 + x + 28$$

(a) plot the given f(x) for $x = 0$ *to* 5 . Use *fsolve* command to find its real root in the interval of $x = 2$ *to* 5 . **(4 marks)**

(b) Transform $f(x) = 0$ into the form $x = g(x)$. Compute a solution of $f(x) = 0$ by fixed-point iteration method, starting from $x_0 = 5$ and doing 5 iterations. Compute the percentage difference of the root found here with the one obtained in (a).

(7 marks)

(c) Compute a solution of $f(x) = 0$ by Newton's method, starting from $x_0 = 5$ and doing 5 iterations. Compute the percentage difference of the root found here with the one obtained in (a). **(7 marks)**

(d) Compute a solution of $f(x) = 0$ by Secant method, starting from $x_0 = 5$ and $x_1 = 5.1$ and doing 5 iterations. Compute the percentage difference of the root found here with the one obtained in (a). **(7 marks)**

Question two

(a) Given a set of data as $f_0 = f(x_0) = f(1) = 2.8$, $f_1 = f(x_1) = f(2) = -0.9$,

$f_2 = f(x_2) = f(3) = 0.3$, $f_3 = f(x_3) = f(4) = 12.8$ and $f_4 = f(x_4) = f(5) = 21.5$,

(i) use the Newton's divided difference interpolation to find its interpolated $p_4(x)$,

(7 marks)

(ii) If using a polynomial of power 3 , i.e., $f(x) = k_0 + k_1\, x + k_2\, x^2 + k_3\, x^3$, to interpolate the given data , determine the values of k_0 , k_1 , k_2 *and* k_3 by the method of least squares. **(7 marks)**

(b) Given the following system of linear equations as

$$\begin{cases} x - 2\,y - 8\,z = 9 \\ 3\,x + 9\,y - 2\,z = 21 \\ 7\,x - y + 4\,z = -65 \end{cases}$$

(i) use *linsolve* command to find the solutions of x , y *and* z, **(3 marks)**

(ii) apply the Gauss-Seidel iteration (5 steps) to the given system, choosing the appropriate pivoting and starting from $x_0 = 1$, $y_0 = 1$ *and* $z_0 = 1$, and compute the iterated solutions of the system. Compare these values with the solutions obtained in (b) (i) and compute their respective percentage differences.

(8 marks)

Question three

(a) Given the following definite integral $\int_0^4 \left(10\,x\,e^{-x} - x^2 \sin(x) + 2\right) d\,x$,

(i) divide the integration range into **ten** equal intervals and compute the value of the given integral by Simpson's rule. **(6 marks)**

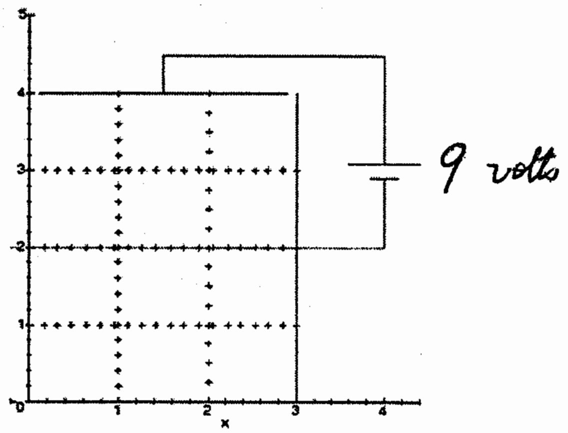
(ii) divide the integration range of $(0 \le x \le 4 \quad \& \quad 0 \le y \le 15)$ into (10×10) equal mesh intervals and use Monte Carlo method with 500 picks to compute the approximate value of the given integral. Compare this result with that obtained in (a)(i) to find their percentage difference. **(9 marks)**

(b) Given the following differential equation $\dfrac{d\,y(x)}{d\,x} = \dfrac{2\sqrt{y(x) - \ln(x)} + 1}{x}$ and boundary conditions of $y(1) = 0 \quad \& \quad x \ge 1$, use Runge-Kutta method by starting with $x = 1 \; and \; h = 0.2$, do 5 steps to find the approximate value of $y(2)$. Compare it with the exact answer of $y(2) = 1.173600195$ to find their percentage difference.

(10 marks)

Question four

An infinite long, rectangular U shaped conducting channel is insulated at the corners from a conducting plate forming the fourth side with interior dimensions as shown below :



The Dirichlet boundary conditions are given as $f(0,y)=0$, $f(3,y)=0$, $f(x,0)=0$ and $f(x,4)=9$ *volts* .

(a) Use the discrete Laplace equations , i.e.,

$$f(i,j)=\frac{f(i-1,j)+f(i+1,j)+f(i,j-1)+f(i,j+1)}{4} \quad \text{where} \quad \begin{matrix} i=1,2 \\ j=1,2,3 \end{matrix},$$

and apply ADI method to find the approximated values of f(1,1) , f(1,2) , f(1,3) , f(2,1) , f(2,2) and f(2,3) . **(10 marks)**

(b) Assign the values of f(1,1) , f(1,2) , f(1,3) , f(2,1) , f(2,2) and f(2,3) all as 4 . Use the renumeration scheme and do 6 rounds of renumeration to find their approximate values. Compare the value of f(2,3) after 6 rounds of renumeration to that obtained in (a) to find their percentage difference. **(15 marks)**.

Question five

(a) Given the following function of x and y as :

$$f(x,y)=\frac{52}{25}x^2+\frac{72}{25}x\,y+\frac{73}{25}y^2-16\,x-18\,y+28\ ,$$

(i) find the maximum value of f and the position of (x , y) that the maximum happens by solving $\frac{\partial f}{\partial x}=0$ *and* $\frac{\partial f}{\partial x}=0$, **(3 marks)**

(ii) use the method of steepest descent , starting with the points $x_0=1$ *and* $y_0=1$, do 5 steps to find the approximate maximum value of f and its approximate (x , y) position . Compare these values with those obtained in (a) (i) to find their respective percentage differences. **(8 marks)**

(b) Given the following function of x and y as :

$f(x,y)=12\,x+25\,y$ where both x and y are positive variables and are subjected to the following constrains :

$-2\,x+y\le 12$ *and* $4\,x+y\le 36$,

(i) plot the constrained region for $x=0$ *to* 9 , **(4 marks)**

(ii) use the Simplex method to find the localized maximum value of f and the position of (x , y) such that this localized maximum occurs. **(10 marks)**