UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION, JULY 2009

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

TITLE OF PAPER: NUMERICAL ANALYSIS

COURSE CODE: E472

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

1.   Answer ANY FOUR out of the five questions.

2.   Each question carries 25 marks.

3.   Marks for different sections are shown in the right-hand margin.

4.   Students are permitted to use MAPLE to answer the questions

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E472 Numerical Analysis

Question one

Given a polynomial function of x as

$$f(x) = x^3 - 5x^2 - 1.01x + 1.88$$

(a) plot the given f(x) for $x = -2$ *to* 8 . Use *fsolve* command to find its real root

in the interval of $x = 3$ *to* 8 . **(4 marks)**

(b) Transform $f(x) = 0$ into the form $x = g(x) = \dfrac{5x^2 + 1.01x - 1.88}{x^2}$. Compute a

solution of $f(x) = 0$ by fixed-point iteration method, starting from $x_0 = 4$ and doing

5 iterations. Compute the percentage difference of the root found here with the one

obtained in (a). **(7 marks)**

(c) Compute a solution of $f(x) = 0$ by Newton's method, starting from $x_0 = 4$ and

doing 5 iterations. Compute the percentage difference of the root found here with the

one obtained in (a). **(7 marks)**

(d) Compute a solution of $f(x) = 0$ by Secant method, starting from $x_0 = 4$ and

$x_1 = 4.1$ and doing 5 iterations. Compute the percentage difference of the root found

here with the one obtained in (a). **(7 marks)**

Given the following 3 by 3 matrix as :

$$A = \begin{pmatrix} -9.2 & -1.6 & 1.5 \\ 0 & 4.3 & 0.7 \\ 6 & 1.3 & 3.7 \end{pmatrix}$$

(a)   find its three real eigenvalues ,   (3 marks)

(b)   use power method and do it for 5 steps to find the numerical value of one of the

eigenvalues of A . Compare it with the corresponding eigenvalue found in (a)

and find their percentage difference.   (8 marks)

(c)   use Liao's extended power method to find the numerical values of the remaining two

real eigenvalues of A . Compare them with the corresponding eigenvalues found in

(a) and find their respective percentage differences.   (14 marks)

Question three

(a)   Given $f(0) = 3$, $f'(0) = 1$, $f(2) = 5$, $f(4) = 31$, $f'(4) = 21$,

find the cubic spline interpolation of $f(x)$ and plot it for $x = 0$ to 4 .

(**10 marks**)

(b)   Given the following definite integral $\int_0^1 \cos(x^2) dx$,

(i)   divide the integration range into **ten** equal intervals and compute the value of

the given integral by the trapezoidal rule.   (**6 marks**)

(ii)   divide the integration range of $(0 \le x \le 1 \ \& \ 0 \le y \le 1)$ into (10×10) equal

mesh intervals and use Monte Carlo method with   800   picks to compute the

approximate value of the given integral. Compare this result with that obtained in

(b)(i) to find their percentage difference.   (**9 marks**)

Question four

(a)   Given the following system of linear equations as

$$\begin{cases} x + 9\,y - 2\,z = 36 \\ 2\,x - y + 8\,z = 121 \\ 6\,x + y + z = 107 \end{cases}$$

(i)   use *linsolve* command to find the solutions of x, y *and* z,   (**3 marks**)

(i)   apply the Gauss-Seidel iteration (6 steps) to the given system, choosing the

appropriate pivoting and starting from $x_0 = 1$, $y_0 = 1$ *and* $z_0 = 1$, and compute

the iterated solutions of the system. Compare these values with the solutions

obtained in (b) (i) and compute their respective percentage differences.

(**8 marks**)

(b)   Given the differential equation $\dfrac{d^2\, y(x)}{d\,x^2} = x\,\dfrac{d\,y(x)}{d\,x} - 3\,y(x)$ with initial conditions of

$$y(0) = 0 \quad \& \quad \left.\dfrac{d\,y(x)}{d\,x}\right|_{x=0} = -3 \quad ,$$

(i)   use *dsolve* command to find its specific solution of $y(x)$. Also find the value

of $y(0.25)$,   (**4 marks**)

(ii)   use Euler's method with $h = 0.05$ and do 5 steps to find the approximate

value of $y(0.25)$. Compare it with that obtained in (b)(i) to find their percentage

difference.   (**10 marks**)