UNIVERSITY OF SWAZILAND

MAIN EXAMINATION, SECOND SEMESTER MAY 2010

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

TITLE OF PAPER:	SIGNALS I
COURSE CODE:	E342
TIME ALLOWED:	THREE HOURS

INSTRUCTIONS:

1. **There are five questions in this paper. Answer any FOUR questions. Each question carries 25 marks.**
2. **If you think not enough data has been given in any question you may assume any reasonable values.**

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THIS PAPER CONTAINS SIX (6) PAGES INCLUDING THIS PAGE

QUESTION 1

A) Explain clearly the difference between

i) Continuous-time and discrete-time signals.

ii) Periodic and non periodic signals

iii) Energy and power signals

iv) Deterministic and random signals

(8 marks)

B) i) State under what conditions will the replacement of a continuous signal with a discrete signal be adequate?

ii) What is a delta function? List the main properties of the delta function

iii) By using a property of the delta function $\delta(t)$, evaluate the integral

$$\int_{-\infty}^{\infty} f_1(t) \times f_2(t)dt, \text{ where } f_1(t) = 2\sin(200\pi t) \text{ and } f_2(t) = \delta(t - 0.25 \times 10^{-3})$$

(7 marks)

C) (i) Define the unit-sample sequence $\delta[n]$ and show that it can be used to express any sequence as the sum of scaled and delayed unit-sample sequences.

(ii) Express the sequence given by $x[n] = 1,1,1,1 0,0,0,.....$ in terms of the unit-sample sequence

(5 marks)

D) Find

i) the total energy associated with the exponentially decaying pulse

$$v(t) = \begin{cases} Ve^{\frac{-t}{\tau}} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

ii) the average power associated with the sinusoidal signal $v(t) = V\cos 2\omega t$

(5 marks)

QUESTION 2

The equations for a number of signals are given below. Determine if they are

a) Periodic or non periodic signals and Energy or power signals

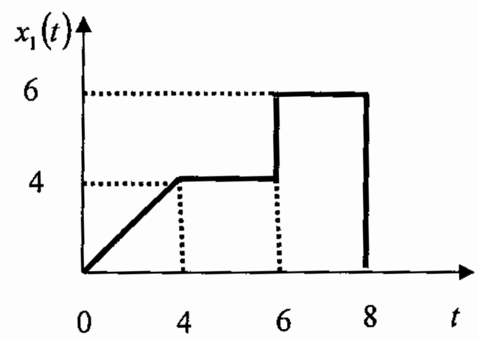
i) $x(t) = 20 \cdot e^{-2t} \cos 100t$; ii) $x(t) = 20 \cdot \cos 2t + 10 \sin 3\pi t$

iii) $x(t) = 30 \cdot \cos 4\pi t[u(t) - u(t-1)]$; iv) $x(t) = 4 \cdot \cos 4\pi t + 6 \sin 5\pi t$

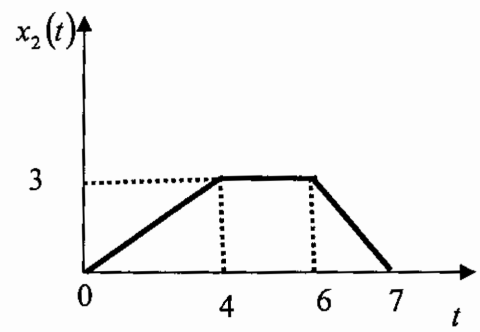
v) $x(t) = 10 \cdot e^{3t} u(t)$; vi) $x(t) = (4 \cdot \cos 4\pi t + 6 \sin 5\pi t) \cdot u(t)$

(12 marks)

b) Express each of the signals shown in **figure 1** a) and b) below in terms of singularity functions. **(5 marks)**



a



b

Figure 1

c) Give the mathematical model for the signal represented in **figure 2** **(4 marks)**

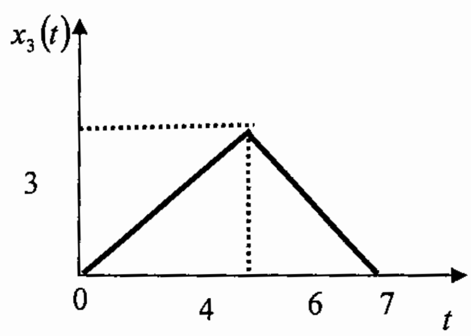


Figure 2

d) Consider a rectangular pulse signal of height A and duration T centered at a point in time $t = t_0 > T$. Sketch the signal waveform in the time domain and obtain an analytic representation in terms of the $rect(\ )$ function. **(4 marks)**

QUESTION 3

a) i) Define the terms signal sampling and sampling period **(4 marks)**

ii) State the sampling theorem and explain the term aliasing error. **(4 marks)**

b)

i) If a sinusoidal of frequency f_0 is sampled at the instants $t = nT$, $n = 0, \pm 1, \pm 2, ...$ the sample sequence can be written as $x[n] = \sin(2\pi f_0 nT) = \sin\left(2\pi \cdot \frac{f_0}{f_s} n\right)$. Show that if $x[n]$ is to be strictly periodic with integer period N then $\frac{f_0}{f_s} = \frac{k}{N}$, where k is an integer. **(5 marks)**

ii) A low pass filter signal $x(t)$ with a bandwidth of $50\ Hz$ is sampled at the Nyquist rate and the resulting sampled values are

$$x(nT_s) = \begin{cases} -1, & -4 \leq n < 0 \\ 1, & 0 < n \leq 4 \\ 0, & otherwise \end{cases}$$

1. Find $x(0.005)$
2. Is this signal power-type or energy-type? Find its power or energy content.

(5marks)

iii) Can a signal be both Power-type and energy-type? Explain your answer.

(2marks)

iv) Samples are to be taken from a record of a continuous-time signal of duration $100\ ms$. The signal contains sinusoidal components with frequencies up to $250\ Hz$. Determine the minimum number of samples that would be sufficient to give a complete representation of the signal. **(5 marks)**

QUESTION 4

a) The peak to average power ratio (PAR) of a given signal $r(t)$ is defined as the ratio of the peak power of $r(t)$ to its average power: $PAR = 10\log_{10}\left(\frac{P_{peak}}{P_{avg}}\right)$.

Find the PAR for a single tone signal $r(t) = A\cos(2\pi F_c t)$.

(7 marks)

b) Consider the signal $r(t) = A\cos(2\pi F_1 t) + B\cos(2\pi F_2 t)$. Let the tones be harmonically related, that is $F_2 = MF_1$.

Find the

i) Peak power of $r(t)$.

ii) Average power of $r(t)$.

iii) PAR.

(12 marks)

c) i) What will be the value of PAR when the powers of the two tones in b) are equal?

ii) What will be the value of PAR in the case of N tones with equal powers

(6 marks)

QUESTION 5

a) Define the following:

i) Pulse duration

ii) Pulse repetition frequency (p.r.f)

(4 marks)

b) A pulse waveform is presented in **figure 3** below:

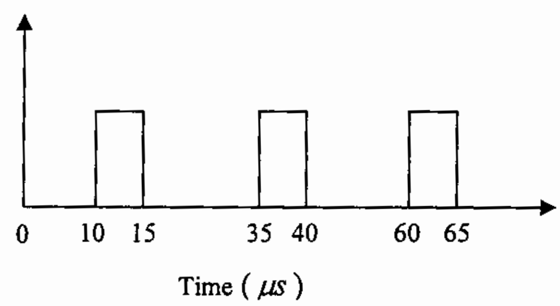


Figure 3

i) Describe the pulse waveform in figure 3.

ii) Without drawing the spectrum of the waveform, determine the lopes spacing

iii) Without drawing the spectrum of the waveform, determine the spectral lines spacing

iv) Calculate the number of spectral lines in each lope.

v) Sketch the spectrum of the waveform

(9 marks)

c) Two voltage signals are modeled as the sinusoids $v_1(t) = 5\cos(3t + 0.5)$ and $v_2(t) = 3 + \cos 2t + 3\sin(4t + \pi/4)$

i) Express the signals $v_1(t)$ and $v_2(t)$ in terms of exponential frequency components

(8 marks)

ii) Sketch the frequency domain representation of the signal $v_1(t)$ **(4 marks)**