UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRICAL & ELECTRONIC ENGINEERING

MAIN EXAMINATION MAY 2010

TITLE OF PAPER: **LINEAR SYSTEMS**

COURSE CODE: **E352**

TIME ALLOWED: **THREE** (3) HOURS

INSTRUCTIONS:

1. Answer question **one** and any other **three** questions.
2. Question one carries 40 marks.
3. Questions 2, 3, 4, and 5 carry 20 marks each.
4. Marks for different sections are shown in the right-hand margin

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This paper contains six (6) pages including this page

Question 1

(A) The relationship between the input x and the output y for a nonlinear system is given by the equation $y = x^2 + 2x$.

At an operating point $x_0 = 2$

(i) obtain an approximate linear equation representing this system, and

(ii) if x changes by +2% what is the change in y, the value of y using the linear equation, and the error in the value of y (13 marks)

(B)The transfer function of a linear system is $\frac{Y(s)}{R(s)} = \frac{2(s+25)}{s^2+15s+50}$

If the input is a unit step,

(i)determine the response y(t) (6 marks)

(ii) calculate the steady state error (1 marks)

(C) For an electromechanical physical system shown in **Figure 1**, the generator which is driven at a constant speed provides the field voltage for the motor and the motor has an inertia J_m and bearing friction b_m. Obtain the transfer function $\frac{\theta_L(s)}{V_f(s)}$ (20 marks)

Note: $T_m = k_m I_g(s)$ and $V_g(s) = k_g I_f(s)$

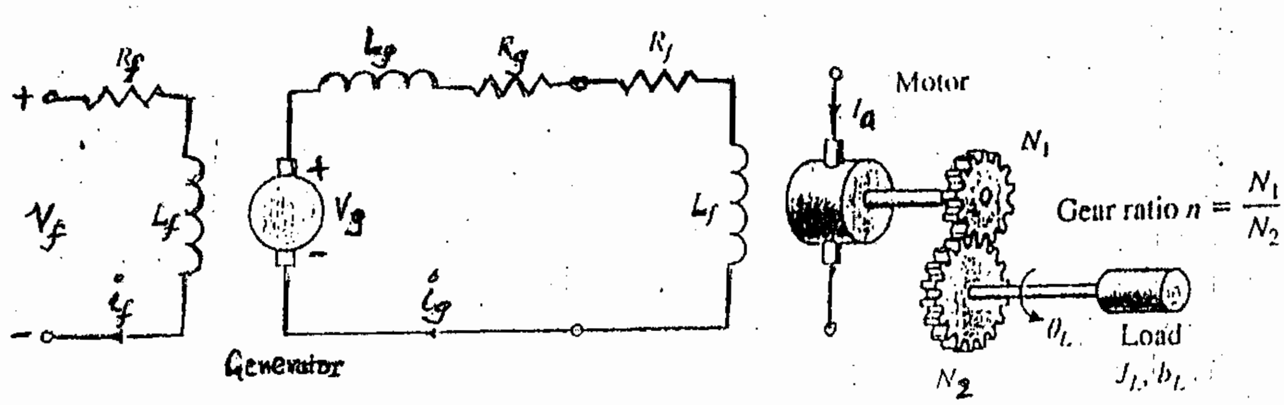


Figure 1

Question 2

Determine the differential equations that describe the physical system shown in **Figure 2** and rewrite the equations in state variable matrix. The state variables $x_1=i$, $x_2=v_1$ and $x_3=v_2$. The input variable is v and the output variable is v_o.

(20 marks)

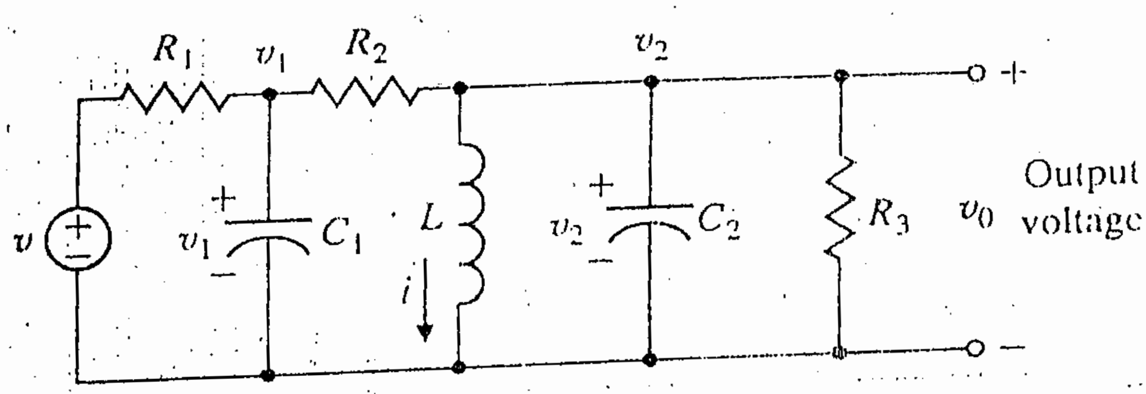


Figure 2

Question 3

A continuous time-invariant linear system is presented by the following model

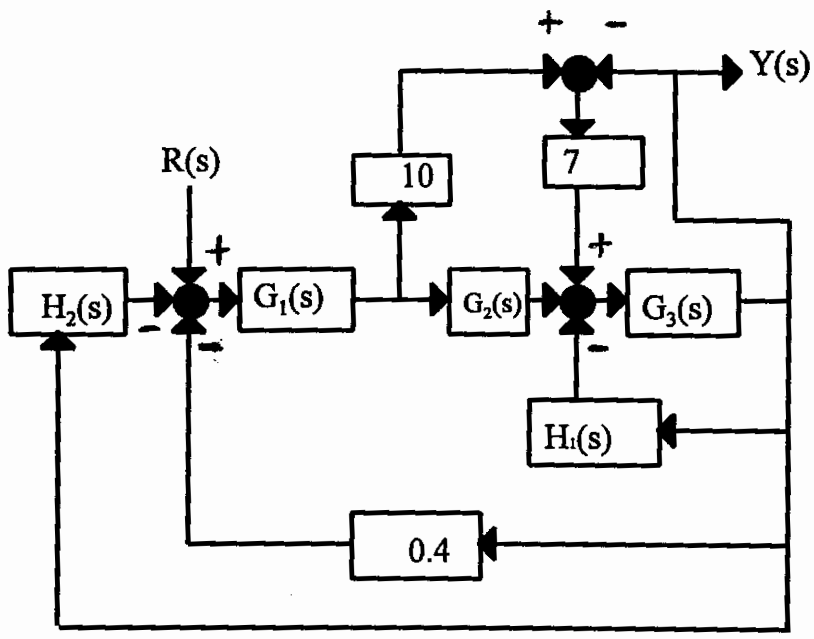
$$\dot{x} = \begin{bmatrix} 1 & 2 & 0 \\ -7 & -2 & -36 \\ -1 & 0 & -7 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 4 & 1 & 2 \end{bmatrix} x$$

Obtain the diagonal form realization of this system.. [20 marks]

Question 4

Use **Mason's gain rule** to find the transfer function of a fuel-injection engine system model whose block diagram is shown in **Figure 4**. (20 marks)



Where $G_1(s) = G_2(s) = G_3(s) = \dfrac{1}{s+5}$,

$$H_1(s) = \frac{0.1}{s} \text{ and } H_2(s) = 0.1S$$

Figure 4

Question 5

A two tank-system has a transfer function $\frac{Y(s)}{R(s)} = T(s) = \frac{1}{s^3 + 10s^2 + 31s + 30}$.

Draw a signal flow graph and determine the state variable equations in a matrix form using

A) the phase variable representation

B) the input feedforward representation.

Figure 5

Partial Tables of z-and s-Transforms

	f(t)	F(s)	F(z)	f(kt)
1.	u(t)	$\frac{1}{s}$	$\frac{z}{z-1}$	u(kT)
2.	t	$\frac{1}{s^2}$	$\frac{Tz}{(z-1)^2}$	kT
3.	t^n	$\frac{n!}{s^{n+1}}$	$\lim_{a\to 0}(-1)^n \frac{d^n}{da^n}\left[\frac{z}{z-e^{-aT}}\right]$	$(kT)^n$
4.	e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$	e^{-akT}
5.	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$	$(-1)^n \frac{d^n}{da^n}\left[\frac{z}{z-e^{-aT}}\right]$	$(kT)^n e^{-akT}$
6.	$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$	$\frac{z\sin\omega T}{z^2-2z\cos\omega T+1}$	$\sin(\omega kT)$
7.	$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$	$\frac{z(z-\cos\omega T)}{z^2-2z\cos\omega T+1}$	$\cos(\omega kT)$
8.	$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$	$\frac{ze^{-aT}\sin\omega T}{z^2-2ze^{-aT}\cos\omega T+e^{-2aT}}$	$e^{-akT}\sin(\omega kT)$
9.	$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$	$\frac{ze^{-aT}\sin\omega T}{z^2-2ze^{-aT}\cos\omega T+e^{-2aT}}$	$e^{-akT}\cos(\omega kT)$
10			$\frac{z}{z+a}$	$a^k\cos(k\pi)$

z-Transform Theorems

	Name	Theorem
1.	Linearity theorem	$z\{af(t)\} = aF(z)$
2.	Linearity theorem	$z\{f_1(t)+f_2(t)\}=F_1(z) + F_2(z)$
3.	Complex differentiation	$z\{e^{-at}f(t)\}=F(e^{aT}z)$
4.	Real translation	$z\{f(t-nT)\}=z^{-n}F(z)$
5.	Complex differentiation	$z\{tf(t)\} = -Tz\frac{dF(z)}{dz}$
6.	Initial value theorem	$f(0) = \lim_{z\to\infty}F(z)$ If the limit exists
7.	Final value theorem	$f(\infty) = \lim_{z\to 1}(1-z^{-1})F(z)$ if the limit exists and the system is stable