UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION 2009/2010

TITLE OF PAPER : ORDINARY DIFFERENTIAL EQUATIONS, PROBABILITY AND STATISTICS

COURSE NUMBER : E371

TIME ALLOWED : THREE HOURS

INSTRUCTIONS : ANSWER ANY FOUR OUT OF FIVE QUESTIONS. EACH QUESTION CARRIES 25 MARKS.

MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.

THIS PAPER HAS SIX PAGES, INCLUDING THIS PAGE.

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Question one

Given the following non-homogeneous ordinary differential equation as

$$\frac{d^2 y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 15\, y(t) = 7t^2 - 3t$$

(a) find its particular solution $y_p(t)$, **(8 marks)**

(b) find the general solution $y_h(t)$ for the homogeneous part of the given differential equation, **(4 marks)**

(c) find the general solution $y_g(t)$ for the above given non-homogeneous differential equation, **(3 marks)**

(d) if given initial conditions as $y(0) = 5$ *and* $\left.\frac{dy(t)}{dt}\right|_{t=0} = 8$,

find its specific solution of $y(t)$ and plot it for $t = 0$ *to* 5 . **(10 marks)**

Question two

(a) If the inverse laplace transform of $F(s)$ is $5\sin(3t) - 4t$, utilize the t - shift theorem to find the inverse laplace transform of $F(s) \times e^{-7s}$.

(3 marks)

(b) If the laplace transform of $f(t)$ is $\frac{3s}{s^2 - 6}$, utilize the s - shift theorem to find the laplace transform of $f(t) \times e^{-4t}$.

(3 marks)

(b) Given the following differential equation as

$$\frac{d^2 y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 5y(t) = f(t)$$

where $$f(t) = \begin{cases} 0 & if \quad t \le 0 \\ 3t & if \quad 0 \le t \le 2 \\ -2t + 10 & if \quad 2 \le t \le 5 \\ 0 & if \quad t \ge 5 \end{cases}$$

(i) find the laplace transform of the above given $f(t)$, **(6 marks)**

(ii) if given the initial conditions as $y(0) = 6$ *and* $\left.\frac{dy(t)}{dt}\right|_{t=0} = -3$, find the laplace transform of $y(t)$, **(8 marks)**

(iii) find the specific solution of $y(t)$ through inverse laplace transform of your answer in (b) (ii). Plot this $y(t)$ for $t = 0$ *to* 10. **(5 marks)**

Question three

Given the following differential equation as

$$\left(1-x^2\right)\frac{d^2\,y(x)}{dx^2}-2\,x\,\frac{d\,y(t)}{dt}+20\,y(t)=0$$

set $y(x)=\sum_{n=0}^{\infty} a_n\,x^{n+s}$ *and* $a_0\neq 0$, utilize the power series method and

(a) write down the indicial equations and find the values of s and possibly the value of a_1 , **(6 marks)**

(b) write down the recurrence relation. Set $a_0=1$ and use the recurrence relation to find the values of a_n ($n=2$ *to* 10) for each value of s found in (a). Write down two independent series solutions truncated up to a_{10} term. **(8 marks)**

(c) (i) write the general solution for the above given differential equation, **(2 marks)**

(ii) if given initial conditions as $y(0)=7$ *and* $\left.\frac{d\,y(x)}{dx}\right|_{x=0}=2$, find the specific solution and plot it for $x=0$ *to* 1 . **(9 marks)**

Question four

(a) Given a probability function $f(0) = f(3) = 1/6$, $f(1) = f(2) = 1/3$. Can f has further possible values? Plot f and its distributive probability function in bar charts. **(10 marks)**

(b) (i) Use the random number generator to generate an ensemble of 30 data of x with its values ranging from 15 to 63 , **(3 marks)**

(ii) find the value of its mean value, variance and standard deviation, **(5 marks)**

(iii) using the interval of 10 starting with 14.5 , i.e., (14.5 to 24.5) , (24.5 to 34.5) , ... , (54.5 to 64.5) , to plot its histogram. **(7 marks)**

Question five

(a) Eight fair coins (i.e., head up and tail up has equal probability) are tossed simultaneously ,

(i) find the probability of precisely 3 heads up , **(4 marks)**

(ii) find the probability of at least 4 heads up. **(6 marks)**

(b) If the defect rate for a skew production is 1 out of 100 and one picks up a handful of 500 skews, find the probability of no more than 5 defected skews are been picked up. **(6 marks)**

(c) Given an ensemble of data which follows a normal distribution with its mean value of 9 and a standard deviation of 1.5 , find the confidence range of these data if the confidence level is set as 96% . **(9 marks)**