UNIVERSITY OF SWAZILAND
FACULTY OF SCIENCE
DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

MAIN EXAMINATION 2009

TITTLE OF PAPER: **SIGNALS II**

COURSE CODE: **E462**

TIME ALLOWED: **THREE** HOURS

INSTRUCTIONS:

1. Answer **ANY FOUR** QUESTIONS.
2. Each question carries 25 marks
3. Useful tables are attached at the end of the question paper

This paper has 6 pages including this page

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QUESTION 1

A) i) A signal $y(t)$ has Fourier transform $Y(\omega)$. If another signal $x(t)$ related to $y(t)$ by the relationship $y(t) = x(t+2)$ and has Fourier transform given by:

$$X(\omega) = \frac{10}{4 + j2\omega}$$

Find $Y(\omega)$ **(4marks)**

(ii) A first order linear system is modeled by the frequency response function

$$H(j\omega) = \frac{3}{1 + j4\omega}.$$

Write down an expression for the response of the system to an input signal $x(t) = 5\sin(0.5t + \pi/6)$.

At what frequency will the magnitude of the frequency response function have fallen to $1/\sqrt{2}$ of its low frequency value?

(8marks)

B) Given that $v(t)$ is the inverse Fourier Transform of $V(\omega)$, where $V(\omega) = AT \sin c(\omega T)$. Find the energy E contained in $v(t)$. **(5marks)**

C) Draw the spectra of the following signals:

i) $x_1(t) = 2 + 4\cos(50t + 0.5\pi) + 12\cos(100t - \pi/3)$

ii) $x_2(t) = 4\cos(2\pi(1000)t)\cos(2\pi(750000)t)$.

(8marks)

QUESTION 2

A) $a[n]$ and $b[n]$ are two sequences defined by $a[n] = 2,3,1$ and $b[n] = 1,2,2,1$.

i) Work out the output sequence $y[n]$ that would result if $a[n]$ and $b[n]$ were respectively the input and the unit-sample response of a system.

ii) Show that the output sequence would be unchanged if $b[n]$ was the input and $a[n]$ was the unit-sample response.

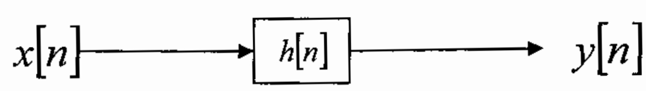
iii) Why does the convolution of a three term sequence with a four term sequence produce a six term sequence?

(6 marks)

B) A linear time invariant (LTI) system is shown below.

i) Explain the term "Linear Time-Invariant" system.

ii) A linear time invariant (LTI) system is shown below.



Given that $x[n] = 2\delta[n-1] - 0.5\delta[n-3]$ and $h[n] = 2\delta[n] + \delta[n-1] - 3\delta[n-3]$,

Find the output sequence **(8 marks)**

C) Given two signals $f_1(t)$ and $f_2(t)$.

i) By using a property of the delta function $\delta(t)$, evaluate the integral

$$\int_{-\infty}^{\infty} f_1(t) \times f_2(t)dt,$$

where $f_1(t) = 2\sin(200\pi t)$ and $f_2(t) = \delta(t - 0.25 \times 10^{-3})$. (2**marks)**

ii) if $f_1(t) = 4\delta(t) + 4\delta(t - 0.025)$ and

$f_2(t) = 4\delta(t + 0.025) + 4\delta(t) + 4\delta(t - 0.025)$,

Obtain the convolution of $f_1(t)$ and $f_2(t)$. (5**marks)**

iii) Causal signals $a[n] = n$ and $b[n] = 2^n$ are applied to a summation block. Write down the values of the first few terms in the output. (2**marks)**

iv) The unit step sequence is defined to be the causal sequence $u[n]=1$ for $n \geq 0$. Plot the sequences $2u[n]$, $u[n-3]$ and $u[n]+u[n-4]$. (2**marks)**

QUESTION 3

A) Compute the Fourier series for the signals in Figure 1 and Figure 2:

i)

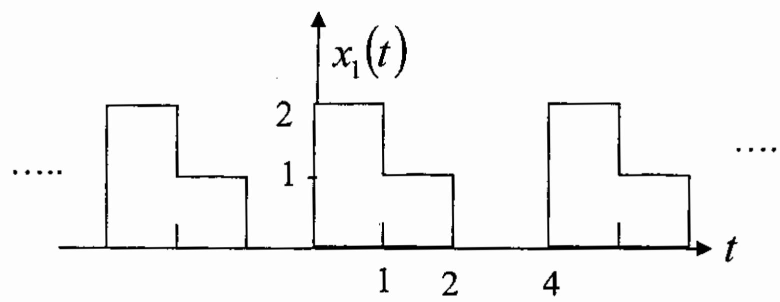


Figure 1

ii)

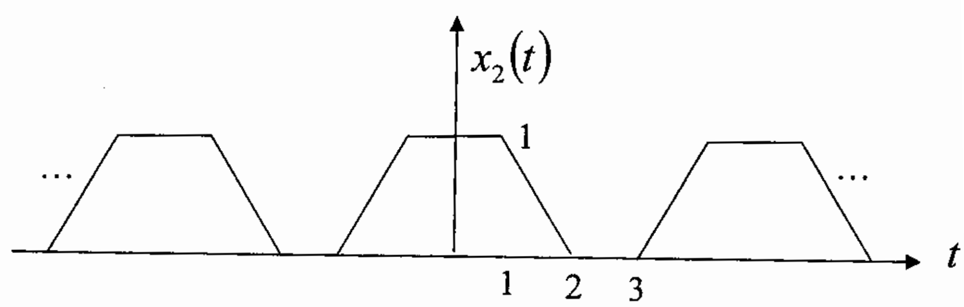


Figure 2

(15 marks)

B) Use the table and find the Fourier Transform of the waveform in Figure 3.

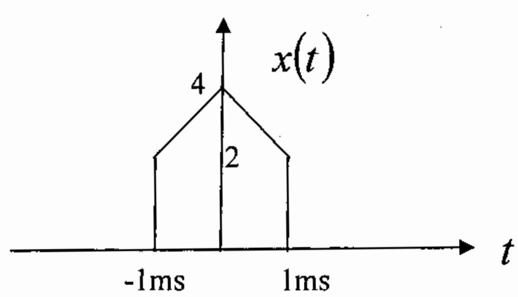


Figure 3 **(5 marks)**

C) A periodic signal in voltage is modeled by the expression

$v(t) = 3 + \cos 2t + 3\sin(4t + \pi/4)$.

Express this signal in exponential form and hence sketch its frequency-domain representation. **(5 marks)**

QUESTION 4

A) Two random variables A and B have the probability density functions given in the figure 4 below. The variables A and B are correlated with correlation coefficient $\rho = 0.4$. Consider a new random variable $Y = A + 2B$

(i) Obtain the mean of Y

(ii) Obtain the variance Y

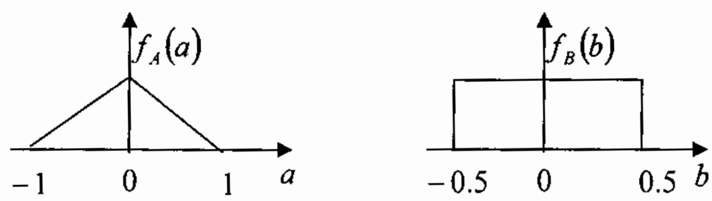


Figure 4

(13 marks)

B) A random variable X is defined by the cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.5x, & 0 < x \leq 1 \\ 0.25x + 0.25, & 1 < x \leq 3 \\ K, & 3 \leq x \end{cases}$$

i) Find the value of K

ii) Find the probability that X lies between 0.5 and 2

iii) Sketch the probability density function of the variable X

(12 marks)

QUESTION 5

A) Determine whether the following are linear or non linear.

i) Ohm's law and

ii) The power dissipated by a resistor as a function of current

(4 marks)

B) The following processing operation is performed by a discrete-time system:

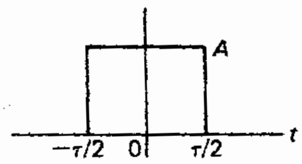
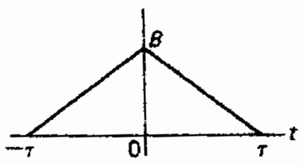
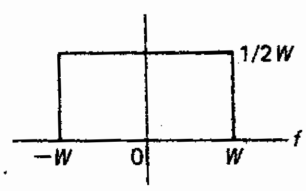
$$y(n) = \frac{1}{M}\sum_{k=0}^{M-1} x(n-k).$$

Show whether the system is

i) linear, ii) time variant, iii) causal or iv) homogenous.

(21 marks)

C.1 Fourier Transform Pairs

Signal	Transform
1.  $\triangleq A\Pi(t/\tau)$	$A\tau \dfrac{\sin \pi f \tau}{\pi f \tau} \triangleq A\tau \operatorname{sinc} f\tau$
2.  $\triangleq B\Lambda(t/\tau)$	$B\tau \dfrac{\sin^2 \pi f \tau}{(\pi f \tau)^2} \triangleq B\tau \operatorname{sinc}^2 f\tau$
3. $e^{-\alpha t}u(t)$	$\dfrac{1}{\alpha + j2\pi f}$
4. $\exp(-\lvert t\rvert/\tau)$	$\dfrac{2\tau}{1 + (2\pi f \tau)^2}$
5. $\exp[-\pi(t/\tau)^2]$	$\tau \exp[-\pi(f\tau)^2]$
6. $\dfrac{\sin 2\pi Wt}{2\pi Wt} \triangleq \operatorname{sinc} 2Wt$	 $\triangleq \dfrac{\Pi(f/2W)}{2W}$
7. $\exp[j(\omega_c t + \phi)]$	$\exp(j\phi)\delta(f - f_c),\ \omega_c = 2\pi f_c$
8. $\cos(\omega_c t + \phi)$	$\frac{1}{2}\delta(f - f_c)\exp(j\phi) + \frac{1}{2}\delta(f + f_c)\exp(-j\phi)$
9. $\delta(t - t_0)$	$\exp(-j2\pi f t_0)$
10. $\sum_{m=-\infty}^{\infty} \delta(t - mT_s)$	$\dfrac{1}{T_s}\sum_{n=-\infty}^{\infty} \delta\left(f - \dfrac{n}{T_s}\right)$
11. $\operatorname{sgn} t = \begin{cases} +1, & t > 0 \\ -1, & t < 0 \end{cases}$	$-\dfrac{j}{\pi f}$
12. $u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$	$\frac{1}{2}\delta(f) + \dfrac{1}{j2\pi f}$
13. $\hat{x}(t)$	$-j \operatorname{sgn}(f) X(f)$