UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRONIC ENGINEERING

MAIN EXAMINATION 2009/2010

TITLE OF PAPER : NUMERICAL ANALYSIS

COURSE NUMBER : E472

TIME ALLOWED : THREE HOURS

INSTRUCTIONS : ANSWER ANY FOUR OUT OF FIVE QUESTIONS. EACH QUESTION CARRIES 25 MARKS.

MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.

STUDENTS ARE PERMITTED TO USE MAPLE TO ANSWER THE QUESTIONS.

THIS PAPER HAS SIX PAGES, INCLUDING THIS PAGE.

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E472 Numerical Analysis

Question one

(a) Given $f(x) = 3x^3 - 24x^2 - 12x + 96$,

(i) use *fsolve* command to find its real roots in the interval of $x = 0$ *to* 10 , **(2 marks)**

(ii) rewrite f(x) = 0 into x = g(x) and use the fixed-point iteration method by doing 5 iterations and starting with $x_0 = 15$ to find one of its roots . Compare this approximate value with one of those exact values found in (a)(i) and compute their percentage difference. **(6 marks)**

(iii) use Newton's method by starting with $x_0 = 15$,and do 5 iterations to find one of its roots . Compare this approximate value with one of those exact values found in (a)(i) and compute their percentage difference. **(6 marks)**

(b) Given the following data as :

$F(2) = 5$, $F(3) = 8$, $F(4) = 10$, $F(5) = 11$, $F(6) = 17$

(i) use Newton's forward divided difference interpolation to find its Lagrange polynomial representation $P_4(x)$, **(6 marks)**

(ii) use least square error extrapolation to find the best straight line fit of the above given data of F . **(5 marks)**

Question two

(a) Given $\int_0^2 e^{-x^2}\,dx$,

(i) use int command to find its value and name it as K , **(2 marks)**

(ii) divide the integration range into 10 equal intervals and use Simpson's rule to find the numerical value of the given integral. Compare it with the value of K and compute their percentage errors , **(6 marks)**

(iii) divide the integration area (x from 0 to 2 & y from 0 to 2) into 10 by 10 mesh intervals (i.e., $h_x = h_y = h = \frac{2}{10}$) , use Monte Carlo method and run 5000 tries to find the numerical value of the given integral. Compare it with the value of K and compute their percentage difference . Make a brief comment on how to reduce their percentage difference . **(8 marks)**

(b) Given the following system of linear equations as :

$$\begin{cases} 2x_1 + 2x_2 + 4x_3 = 6 \\ 13x_1 + 5x_2 + 4x_3 = -43 \\ 10x_1 + 43x_2 + 14x_3 = 78 \end{cases}$$

(i) use *linsolve* command to find the solutions of x_1 , x_2 & x_3 , **(2 marks)**

(ii) use iteration method , starting with $x_1 = 1$, $x_2 = 1$ & $x_3 = 1$ and do 10 iterations , to find their numerical answers. Compare these values with those answers found in (b)(i) and compute their respective percentage errors . **(7 marks)**

Question three

(a) Given the following 3 by 3 matrix as

$$A=\begin{pmatrix} 6 & 1 & 2 \\ 2 & 5 & -4 \\ 5 & -2 & 7 \end{pmatrix}$$

(i) use *eigenvals* command to find its eigenvalues , **(2 marks)**

(ii) use Liao's extended power method to find the numerical values of its real eigenvalues . Compare them with those obtained in (a)(i) and compute their respective percentage errors . (Note : do 5 times for each do loop)
(12 marks)

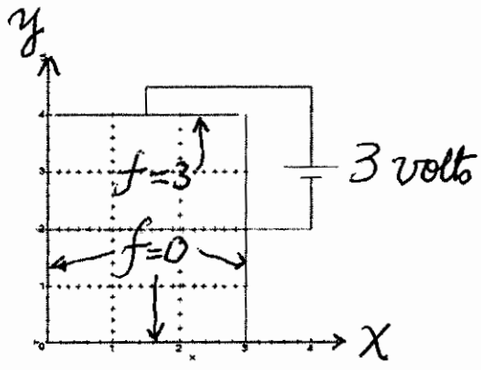
(b) Given $\frac{d\,y(x)}{d\,x}=\frac{2\sqrt{y(x)-\ln(x)}+1}{x}$, $y(1)=0$ & $x\geq 1$, its specific solution can be shown to be $y(x)=(\ln(x))^2+\ln(x)$ and the exact value of y(2) is 1.173600195 ,

(i) use Euler's method , starting with x = 1 and taking $h=0.2$ and do 5 steps , to find the approximate value of y at x = 2 . Compare this approximate value with the given exact value of y(2) and calculate their percentage difference.
(6 marks)

(ii) use Runge-Kutta method , starting with x = 1 and taking $h=0.2$ and do 5 steps , to find the approximate value of y at x = 2 . Compare this approximate value with the given exact value of y(2) and calculate their percentage difference.
(7 marks)

Question four

(a) An infinite long, rectangular U shaped conducting channel is insulated at the corners from a conducting plate forming the fourth side with interior dimensions as shown below :



The Dirichlet boundary conditions are given as $f(0,y)=0$, $f(3,y)=0$, $f(x,0)=0$ and $f(x,4)=3$ *volts* .

(i) Use the discrete Laplace equations , i.e.,

$$f(i,j)=\frac{f(i-1,j)+f(i+1,j)+f(i,j-1)+f(i,j+1)}{4} \quad \textit{where} \quad \begin{matrix} i=1,2 \\ j=1,2,3 \end{matrix},$$

and apply ADI method to find the approximated values of f(1,1) , f(1,2) , f(1,3) , f(2,1) , f(2,2) and f(2,3) . **(7 marks)**

(ii) Assign the values of f(1,1) , f(1,2) , f(1,3) , f(2,1) , f(2,2) and f(2,3) all as 1 . Use Liao's renumeration scheme and do 5 rounds of renumeration to find their approximate values. Compare the value of f(2,3) after 5 rounds of renumeration to that obtained in (a)(i) to find their percentage difference. **(9 marks)**

(b) Given the following $f=3x^2+4xy+y^2+7y+15$, find the extremum value of the given f and at what point of (x,y) this extremum value happens by

(i) using $\frac{\partial f}{\partial x}=0$ & $\frac{\partial f}{\partial y}=0$, **(2 marks)**

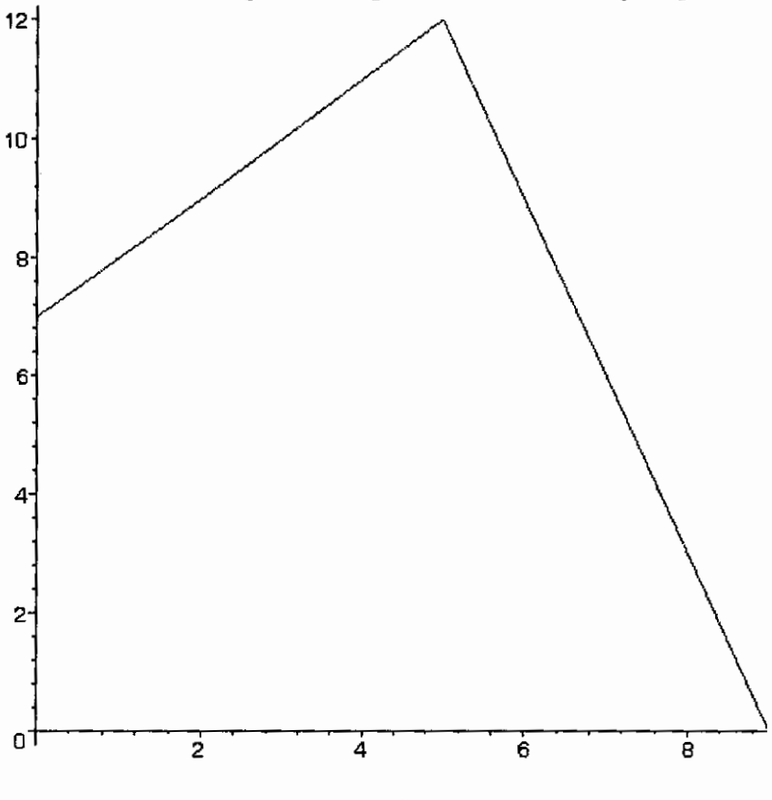
(ii) using the numerical method of steepest descent (starting at the point (x = 3 , y = 5) and do 5 steps of descent). Compare the answers here with those obtained in (b)(i) and calculate their respective percentage differences. **(7 marks)**

Question five

Q.5 Given the following $f = 36\,x_1 + 22\,x_2$ where the positive variables x_1 & x_2 are under the following constraints:

$$\begin{cases} -x_1 + x_2 \le 7 \\ 3\,x_1 + x_2 \le 27 \end{cases}$$

(a) (i) plot the constrained region and reproduce the following diagram



(Hint : the intersection points happens at $(x_1 = 5\,,\, x_2 = 12)$) **(4 marks)**

(ii) use Liao's "pick and compare" method, divide the region in (a)(i) into 100 by 100 meshes , find the approximate maximum value of f and at which values of x_1 & x_2 this maximum value happens. **(9 marks)**

(b) use Simplex method , construct a 3 by 6 matrix and introducing two slack variables , find the maximum value of f and at which values of x_1 & x_2 this maximum value happens. Compare them with those obtained in (a)(ii) and compute their respective percentage differences.
(Hint : choose appropriate pivoting rows for add-rows and ensure the coefficients of the slack variables of the final evolved first row equation are having positive values.)
(12 marks)