University of Swaziland
Faculty of Science
Department of Electrical and Electronic Engineering

Main Examination 2010

Title of Paper: Signals II

Course Number: E462

Time Allowed: 3 hrs

Instructions:

1. Answer any four (4) questions.
2. Each questions carries 25 marks.
3. Useful tables are attached at the end of the question paper

This paper should not be opened until permission has been given by the invigilator.

This paper contains eight (8) pages including this page.

Question 1

(a) Find the frequency response of the system determined by the input/output relationship:

$$\frac{d^2y(t)}{dt^2}+7\frac{dy(t)}{dt}+12y(t)=\frac{dx(t)}{dt}+2x(t)$$

and determine its impulse response, $h(t)$. (10 marks)

(b) The signal $g(t)$ has the Fourier transform:

$$G(\omega)=\frac{j\omega}{-\omega^2+5j\omega+6}$$

Find the Fourier transforms of

(i) $g(2t)$ (3 marks)

(ii) $g(3t-6)$ (3 marks)

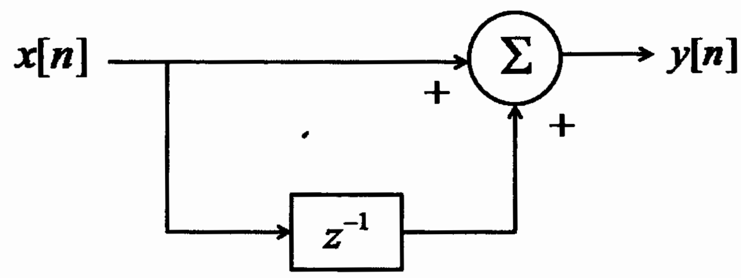
(iii) $g(-t)$ (3 marks)

(iv) $\frac{dg(t)}{dt}$ (3 marks)

(v) $e^{-j100t}g(t)$ (3 marks)

Question 2

Consider the discrete-time LTI system shown below:



(a) Write down the expression that relates $y[n]$ to $x[n]$. (2 marks)

(b) Find the frequency response $H(\Omega)$ of the system. (5 marks)

(c) Find the impulse response $h[n]$ of the system. (3 marks)

(d) Find the magnitude spectrum, $|H(\Omega)|$ and the phase spectrum, $\theta(\Omega)$. (5 marks)

(e) Find and plot the Fourier transform of $x[n]=1$. (2 marks)

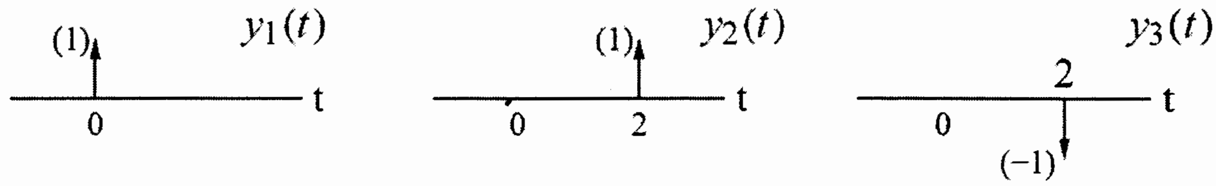
(f) Find the Fourier transform of $x[n]=-a^n u[-n-1]$, a real. (3 marks)

(g) A discrete-time LTI system is described by $y[n]-\frac{3}{4}y[n-1]+\frac{1}{8}y[n-2]=x[n]$ where $x[n]$ and $y[n]$ are then input and output of the system, respectively. Determine the frequency response $H(\Omega)$ of the system.

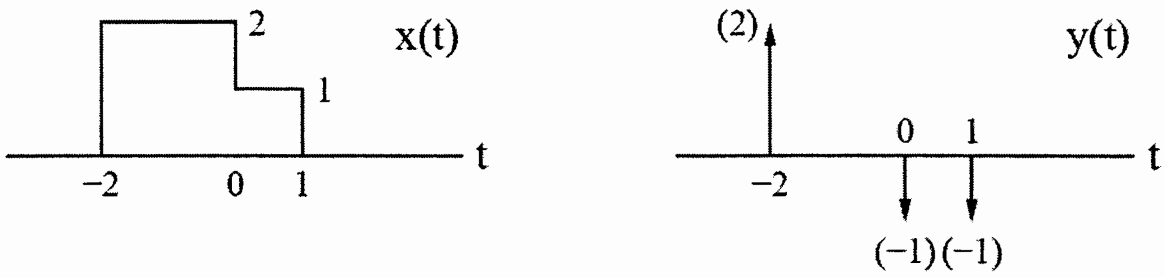
(5 marks)

Question 3

(a) Suppose $y_1(t)$, $y_2(t)$ and $y_3(t)$ are as shown below:



If $x(t)$ and $y(t)$ are:



Then sketch

(i) $x(t) * y_1(t)$, (3 marks)

(ii) $x(t) * y_2(t)$, (3 marks)

(iii) $x(t) * y_3(t)$, (3 marks)

(iv) $y(t) * y_1(t)$, (3 marks)

(v) $y(t) * y_2(t)$, and (3 marks)

(vi) $y(t) * y_3(t)$. (3 marks)

(b) Perform the cross-correlation $R_{xy}[k]$ of the following periodic sequences:
$x[n] = [2,\ 1,\ 3,\ 0]$ and $h[n] = [3,\ 2,\ 4,\ 3]$. (5 marks)

c) Define autocorrelation. (2 marks)

Question 4

(a) Suppose a random variable has the following CDF:

$$F_x(x) = \begin{cases} 0, \; x \leq -10 \\ 0.01(x+10)^2, \; -10 < x \leq 0 \\ 1, \; x > 0 \end{cases}$$

(i) Plot $f_x(x)$, the PDF. (5 marks)

(ii) Find $P[-5 < X \leq 5]$. (5 marks)

(iii)Find the expected value of X. (5 marks)

(b) Given the following probability density function:

$$f_x(t) = \begin{cases} \frac{1}{2}t, \; 0 < t < 2 \\ 0, \; otherwise \end{cases}$$

(i) Calculate the mean. (5 marks)

(ii) Calculate the variance. (5 marks)

Question 5

(a) A linear time invariant (LTI) system is shown below.

Let $x[n] = 3\delta[n-1] - \delta[n-3]$ and $h[n] = 3\delta[n] + 0.5\delta[n-1] - 2\delta[n-3]$.

Find the output sequence $y[n]$. (8 marks)

(b) Given that the probability density function spent in line by a transistor in a production plant is:

$$f_x(t) = \begin{cases} 0.1e^{\frac{-t}{10}}, & \text{if } t \geq 0 \\ 0, & \text{if } t < 0 \end{cases}$$

where t is the number of minutes spent waiting in line.

(i) Determine the probability that a transistor will be in line for at least 6 minutes. (4 marks)

(ii) Determine the mean waiting time. (5 marks)

(c) A time signal $x(t)$ has the Fourier transform $\dfrac{4}{\left(4+\omega^2\right)}$. Find the Fourier transforms of the following signals:

(i) $x_1(t) = x(2t-3)$ (4 marks)

(ii) $x_2(t) = \int_{-\infty}^{t} x(\tau)d\tau$ (4 marks)

Fourier Transform Properties

Property	Signal	Fourier transform
	$x(t)$	$X(\omega)$
	$x_1(t)$	$X_1(\omega)$
	$x_2(t)$	$X_2(\omega)$
Linearity	$a_1 x_1(t) + a_2 x_2(t)$	$a_1 X_1(\omega) + a_2 X_2(\omega)$
Time shifting	$x(t - t_0)$	$e^{-j\omega t_0} X(\omega)$
Frequency shifting	$e^{j\omega_0 t} x(t)$	$X(\omega - \omega_0)$
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert} X\left(\frac{\omega}{a}\right)$
Time reversal	$x(-t)$	$X(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Time differentiation	$\frac{dx(t)}{dt}$	$j\omega X(\omega)$
Frequency differentiation	$(-jt)x(t)$	$\frac{dX(\omega)}{d\omega}$
Integration	$\int_{-\infty}^{t} x(\tau)\, d\tau$	$\pi X(0)\delta(\omega) + \frac{1}{j\omega} X(\omega)$
Convolution	$x_1(t) * x_2(t)$	$X_1(\omega) X_2(\omega)$
Multiplication	$x_1(t) x_2(t)$	$\frac{1}{2\pi} X_1(\omega) * X_2(\omega)$
Real signal	$x(t) = x_e(t) + x_o(t)$	$X(\omega) = A(\omega) + jB(\omega)$
		$X(-\omega) = X^*(\omega)$
Even component	$x_e(t)$	$\mathrm{Re}\{X(\omega)\} = A(\omega)$
Odd component	$x_o(t)$	$j\,\mathrm{Im}\{X(\omega)\} = jB(\omega)$

Parseval's relations

$$\int_{-\infty}^{\infty} x_1(\lambda) X_2(\lambda)\, d\lambda = \int_{-\infty}^{\infty} X_1(\lambda) x_2(\lambda)\, d\lambda$$

$$\int_{-\infty}^{\infty} x_1(t) x_2(t)\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_1(\omega) X_2(-\omega)\, d\omega$$

$$\int_{-\infty}^{\infty} |x(t)|^2\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2\, d\omega$$

Continuous Time Fourier Transform

$x(t)$	$X(\omega)$
$\delta(t)$	1
$\delta(t - t_0)$	$e^{-j\omega t_0}$
1	$2\pi\delta(\omega)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
$\sin \omega_0 t$	$-j\pi[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$u(-t)$	$\pi\delta(\omega) - \frac{1}{j\omega}$
$e^{-at}u(t), a > 0$	$\frac{1}{j\omega + a}$
$te^{-at}u(t), a > 0$	$\frac{1}{(j\omega + a)^2}$
$e^{-a\lvert t\rvert}, a > 0$	$\frac{2a}{a^2 + \omega^2}$
$\frac{1}{a^2 + t^2}$	$e^{-a\lvert\omega\rvert}$
$e^{-at^2}, a > 0$	$\sqrt{\frac{\pi}{a}}\, e^{-\omega^2/4a}$
$p_a(t) = \begin{cases} 1 & \lvert t\rvert < a \\ 0 & \lvert t\rvert > a \end{cases}$	$2a\frac{\sin \omega a}{\omega a}$
$\frac{\sin at}{\pi t}$	$p_a(\omega) = \begin{cases} 1 & \lvert\omega\rvert < a \\ 0 & \lvert\omega\rvert > a \end{cases}$
$\mathrm{sgn}\, t$	$\frac{2}{j\omega}$
$\sum_{k=-\infty}^{\infty} \delta(t - kT)$	$\omega_0 \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0), \omega_0 = \frac{2\pi}{T}$

Discrete Time Fourier Transform Properties

Property	Sequence	Fourier transform
	$x[n]$	$X(\Omega)$
	$x_1[n]$	$X_1(\Omega)$
	$x_2[n]$	$X_2(\Omega)$
Periodicity	$x[n]$	$X(\Omega + 2\pi) = X(\Omega)$
Linearity	$a_1 x_1[n] + a_2 x_2[n]$	$a_1 X_1(\Omega) + a_2 X_2(\Omega)$
Time shifting	$x[n - n_0]$	$e^{-j\Omega n_0} X(\Omega)$
Frequency shifting	$e^{j\Omega_0 n} x[n]$	$X(\Omega - \Omega_0)$
Conjugation	$x^*[n]$	$X^*(-\Omega)$
Time reversal	$x[-n]$	$X(-\Omega)$
Time scaling	$x_{(m)}[n] = \begin{cases} x[n/m] & \text{if } n = km \\ 0 & \text{if } n \neq km \end{cases}$	$X(m\Omega)$
Frequency differentiation	$nx[n]$	$j\dfrac{dX(\Omega)}{d\Omega}$
First difference	$x[n] - x[n-1]$	$(1 - e^{-j\Omega})X(\Omega)$
Accumulation	$\sum_{k=-\infty}^{n} x[k]$	$\pi X(0)\delta(\Omega) + \dfrac{1}{1 - e^{-j\Omega}} X(\Omega)$, $\lvert\Omega\rvert \le \pi$
Convolution	$x_1[n] * x_2[n]$	$X_1(\Omega) X_2(\Omega)$
Multiplication	$x_1[n] x_2[n]$	$\dfrac{1}{2\pi} X_1(\Omega) \otimes X_2(\Omega)$
Real sequence	$x[n] = x_e[n] + x_o[n]$	$X(\Omega) = A(\Omega) + jB(\Omega)$; $X(-\Omega) = X^*(\Omega)$
Even component	$x_e[n]$	$\text{Re}\{X(\Omega)\} = A(\Omega)$
Odd component	$x_o[n]$	$j\,\text{Im}\{X(\Omega)\} = jB(\Omega)$
Parseval's relations		

$$\sum_{n=-\infty}^{\infty} x_1[n] x_2[n] = \frac{1}{2\pi} \int_{2\pi} X_1(\Omega) X_2(-\Omega)\, d\Omega$$

$$\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi} \int_{2\pi} \lvert X(\Omega)\rvert^2\, d\Omega$$

Discrete Time Fourier Transform

$x[n]$	$X(\Omega)$
$\delta[n]$	1
$\delta[n - n_0]$	$e^{-j\Omega n_0}$
$x[n] = 1$	$2\pi\delta(\Omega)$, $\lvert\Omega\rvert \le \pi$
$e^{j\Omega_0 n}$	$2\pi\delta(\Omega - \Omega_0)$, $\lvert\Omega\rvert, \lvert\Omega_0\rvert \le \pi$
$\cos \Omega_0 n$	$\pi[\delta(\Omega - \Omega_0) + \delta(\Omega + \Omega_0)]$, $\lvert\Omega\rvert, \lvert\Omega_0\rvert \le \pi$
$\sin \Omega_0 n$	$-j\pi[\delta(\Omega - \Omega_0) - \delta(\Omega + \Omega_0)]$, $\lvert\Omega\rvert, \lvert\Omega_0\rvert \le \pi$
$u[n]$	$\pi\delta(\Omega) + \dfrac{1}{1 - e^{-j\Omega}}$, $\lvert\Omega\rvert \le \pi$
$-u[-n-1]$	$-\pi\delta(\Omega) + \dfrac{1}{1 - e^{-j\Omega}}$, $\lvert\Omega\rvert \le \pi$
$a^n u[n]$, $\lvert a\rvert < 1$	$\dfrac{1}{1 - ae^{-j\Omega}}$
$-a^n u[-n-1]$, $\lvert a\rvert > 1$	$\dfrac{1}{1 - ae^{-j\Omega}}$
$(n+1)a^n u[n]$, $\lvert a\rvert < 1$	$\dfrac{1}{\left(1 - ae^{-j\Omega}\right)^2}$
$a^{\lvert n\rvert}$, $\lvert a\rvert < 1$	$\dfrac{1 - a^2}{1 - 2a\cos\Omega + a^2}$
$x[n] = \begin{cases} 1 & \lvert n\rvert \le N_1 \\ 0 & \lvert n\rvert > N_1 \end{cases}$	$\dfrac{\sin\left[\Omega\left(N_1 + \frac{1}{2}\right)\right]}{\sin(\Omega/2)}$
$\dfrac{\sin Wn}{\pi n}$, $0 < W < \pi$	$X(\Omega) = \begin{cases} 1 & 0 \le \lvert\Omega\rvert \le W \\ 0 & W < \lvert\Omega\rvert \le \pi \end{cases}$
$\sum_{k=-\infty}^{\infty} \delta[n - kN_0]$	$\Omega_0 \sum_{k=-\infty}^{\infty} \delta(\Omega - k\Omega_0)$, $\Omega_0 = \dfrac{2\pi}{N_0}$