UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRIC AND ELECTRONIC ENGINEERING

SUPPLEMENTARY EXAMINATION 2010/2011

TITLE OF PAPER	:	NUMERICAL ANALYSIS
COURSE NUMBER	:	E472
TIME ALLOWED	:	THREE HOURS
INSTRUCTIONS	:	ANSWER ANY FOUR OUT OF FIVE QUESTIONS. EACH QUESTION CARRIES 25 MARKS. MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN. STUDENTS ARE PERMITTED TO USE MAPLE TO ANSWER THE QUESTIONS.

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E472 Numerical Analysis

Question one

Given a function of x as

$$f(x) = \cos\left(\frac{x}{3}\right) - e^{-\frac{x}{4}}$$

(a) plot the given f(x) for $x = -2$ *to* 8 . Use *fsolve* command to find its real root in the interval of $x = 1$ *to* 8 . **(4 marks)**

(b) Transform $f(x) = 0$ into the form $x = g(x)$ and compute a solution of $f(x) = 0$ by fixed-point iteration method, starting from $x_0 = 5$ and doing 7 iterations. Compute the percentage difference of the root found here with the one obtained in (a). **(7 marks)**

(c) Compute a solution of $f(x) = 0$ by Newton's method, starting from $x_0 = 5$ and doing 7 iterations. Compute the percentage difference of the root found here with the one obtained in (a). **(7 marks)**

(d) Compute a solution of $f(x) = 0$ by Secant method, starting from $x_0 = 5$ and $x_1 = 5.1$ and doing 7 iterations. Compute the percentage difference of the root found here with the one obtained in (a). **(7 marks)**

Question two

(a) Given the following data of $f(x)$ as :

$f(1)=-3.2$, $f(3)=-1.3$, $f(5)=0.1$ & $f(7)=0.8$

(i) use Newton's forward divided method to find its 3^{rd} order Lagrange polynomial extrapolation , i.e., $P_3(x)$, of the given data of $f(x)$. Then plot $P_3(x)$ for $x=0$ *to* 7 . **(8 marks)**

(ii) set $f(x)=k_1\,x+k_2$ and use the least square error fitting to find the appropriate values of k_1 & k_2 . Plot this extrapolated $f(x)$ as well as the given data of $f(x)$ for $x=0$ *to* 7 and show them in a single display . **(8 marks)**

(b) Given the following 2 by 2 matrix A as $A=\begin{pmatrix}-2 & 4\\ 6 & 8\end{pmatrix}$,

(i) use *eigenvals* command to find its two eigenvalues, **(1 mark)**

(ii) starting with $x_0=\begin{pmatrix}1\\1\end{pmatrix}$ and using power method up to 5 loops , to find one of the numerical eigenvalues of A. Then using the law that the trace of A is invariant before and after similarity transformation, to find the remaining numerical eigenvalue of A. Compare these two numerical eigenvalues of A with those found in (b)(i) and compute their respective percentage errors . **(8 marks)**

Question three

(a) Given the following definite integral $\int_{-1}^{3} e^{-\frac{x^2}{10}}\, dx$,

(i) plot the given integrand for $x = -1$ *to* 3 and use int command to find the value of the given definite integral, **(4 marks)**

(ii) divide the integration range into **ten** equal intervals and compute the value of the given integral by the trapezoidal rule. Compare this value with the one obtained in (a)(i) and compute their percentage difference. **(8 marks)**

(b) Given the following system of linear equations as

$$\begin{cases} x - 4\,y + 8\,z = -30 \\ 5\,x - 2\,y + 5\,z = 45 \\ x - 6\,y + 4\,z = -28 \end{cases}$$

(i) use *linsolve* command to find the solutions of x , y *and* z, **(3 marks)**

(ii) apply the Gauss-Seidel iteration (5 steps) to the given system, choosing the appropriate pivoting and starting from $x_0 = 1$, $y_0 = 1$ *and* $z_0 = 1$, and compute the iterated solutions of the system. Compare these values with the solutions obtained in (a)(i) and compute their respective percentage differences. **(10 marks)**

Question four

(a) Given the differential equation $\dfrac{d\,y(x)}{d\,x} + y(x) + e^{-\frac{x}{4}} = 0$ with initial condition of

$y(0) = 2$,

(i) use *dsolve* command to find its specific solution of $y(x)$. Also find the value of $y(3)$, **(3 marks)**

(ii) use improved Euler's method, i.e., Heun's method, with $h = 0.3$ and do 10 steps to find the approximate value of $y(3)$. Compare it with that obtained in (a)(i) to find their percentage difference . **(8 marks)**

(b) Given the differential equation $\dfrac{d^2\,y(x)}{d\,x^2} = x\,\dfrac{d\,y(x)}{d\,x} + x\,y(x)$ with initial conditions of

$y(0) = 4$ & $\left.\dfrac{d\,y(x)}{d\,x}\right|_{x=0} = -2$,

(i) use *dsolve* command to find its specific solution of $y(x)$. Also find the value of $y(2)$, **(3 marks)**

(ii) use Euler's method with $h = 0.2$ and do 10 steps to find the approximate value of $y(2)$. Compare it with that obtained in (b)(i) to find their percentage difference . **(11 marks)**

Question five

(a) Given the following function of x and y as :

$f(x,y) = x^2 + 2\,x\,y + 5\,y^2 - 6\,x + 8\,y + 20$,

(i) find the maximum value of f and the position of (x , y) that the maximum happens by solving $\frac{\partial f}{\partial x} = 0$ *and* $\frac{\partial f}{\partial y} = 0$, **(3 marks)**

(ii) use the method of steepest descent , starting with the points $x_0 = 1$ *and* $y_0 = 1$, do 5 steps to find the approximate maximum value of f and its approximate (x , y) position . Compare these values with those obtained in (a) (i) to find their respective percentage differences. **(9 marks)**

(b) Given the following function of x and y as :

$f(x,y) = 10\,x + 20\,y$ where both x and y are positive variables and are subjected to the following constrains : $-x + y \le 10$ *and* $3\,x + y \le 30$,

(i) plot the constrained region for $x = 0$ *to* 10 , **(3 marks)**

(ii) use the Simplex method to find the localized maximum value of f and the position of (x , y) such that this localized maximum occurs. **(10 marks)**