UNVERSITY OF SWAZILAND

MAIN EXAMINATION – SEMESTER I - NOV/DEC 2010

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

TITLE OF THE PAPER	**:**	**PROBABILITY AND STATISTICS**
COURSE CODE	**:**	**EE301**
TIME ALLOWED	**:**	**3 HOURS**

INSTRUCTIONS:

1. **ANWSER ANY FIVE OF THE SEVEN QUESTIONS IN THIS PAPER.**
2. **STATISTICAL TABLES ARE ATTACHED AT THE END OF THIS PAPER.**
3. **CALCULATORS MAY BE USED.**

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR

THIS PAPER CONTAINS SIX PAGES INCLUDING THIS PAGE.

Question 1

A Personal Identification Number (PIN) consists of four digits in order, each of which may be any one of 0, 1, 2, …, 9.

a) Find the number of PINs satisfying each of the following requirements.
   (i) All four digits are different.
   (ii) There are exactly three different digits.
   (iii) There are two different digits, each of which occurs twice.
   (iv) There are exactly three digits the same.

(9 Marks)

b) Two PINs are chosen independently and at random, and you are given that each PIN consists of four different digits. Let X be the random variable denoting the number of digits that the two PINs have in common.

   (i) Explain clearly why $P(X = k) = \dfrac{\binom{4}{k}\binom{6}{4-k}}{\binom{10}{4}}$, for $k = 0, 1, 2, 3, 4$.

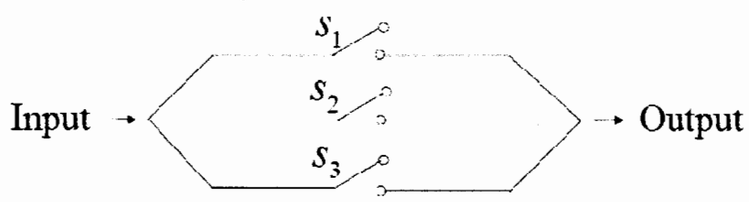
(4 Marks)

   (ii) Hence write down the values of the probability mass function of $\boldsymbol{X}$, and find its mean and variance.

(7 Marks)

Question 2

Three switches connected in parallel operate independently. Each switch remains closed with probability p.



(a) Find the probability of receiving an input signal at the output.

(10 Marks)

(b) Find the probability that switch S_1 is open given that an input signal is received at the output.

(10 Marks)

Question 3

(a) The random variable X has the binomial distribution with probability mass function

$$P(X=x)=\binom{2}{x}p^x(1-p)^{2-x}, \quad x=0,1,2; \quad 0<p<1.$$

Write down E(X), Var(X) and P(X = 2) in terms of the parameter p. Also find P(X = 0 | X < 2) and P(X = 1 | X < 2), simplifying your answers as far as possible.

(12 Marks)

(b) The random variable T has the exponential distribution with rate parameter λ, so that the probability density function (pdf) of T is

$$f_T(t)=\lambda e^{-\lambda t}, \quad t>0, \quad \lambda>0.$$

Obtain the cumulative distribution function (cdf) $F_T(t)$ of T, and Show that $P(a < T \leq b) = e^{-\lambda a} - e^{-\lambda b}$.

(8 Marks)

Question 4

Flaws in lengths of fibre optic cable made by Company A occur in a Poisson process at rate λ_A per metre length, so that the number of flaws X in a length of l metres of rope has the Poisson probability mass function

$$P(X=x)=\frac{\exp(-\lambda_A l).(\lambda_A l)^x}{x!}, \quad x=0,1,2,\ldots; \quad \lambda_A>0.$$

(a) Find the probability that there are (i) no flaws, (ii) more than 2 flaws, in a 1000-metre length of rope made by company A, given that $\lambda_A = 0.002$.

(4 Marks)

(b) Company B makes similar cable, indistinguishable in appearance from that made by Company A, in which flaws occur in a Poisson process at rate $\lambda_B = 0.003$ per metre. A communications system is installed with 100 metres of rope from Company A and 100 metres of rope from Company B. Assuming that the lengths of cable supplied by A and B are independent, find the probability that (i) there are no flaws, (ii) there is exactly one flaw, in the installation.

(5 Marks)

(c) A telecommunications company buys 75% of cable from Company A and 25% from Company B. The supplier's label has become detached from a drum of cable of length 2 km which is found to have 7 flaws. Find the probability that this drum was supplied by Company A.

(6 Marks)

(d) Suppose, instead, that the cable in this drum had been found to have 8 flaws. Find the probability that this drum was supplied by Company A. Compare this probability with your answer to part (c) and comment.

(5 Marks)

Question 5

The joint probability density function of the random variables X and Y is

$$f(x, y) = \frac{1}{2\pi} \exp\left(\frac{1}{4}(x \quad 1)^2 \quad (y \quad \frac{1}{4}(1 + x))^2 \right), \quad \infty < x < \infty, \quad \infty < y < \infty.$$

(a) Show that X has the Normal distribution with mean 1 and variance 2.

(7 Marks)

(b) Show that the moment generating function of X is $m_X(t) = \exp(t + t^2)$

(7 Marks)

(c) Use the moment generating function to find $E(X^3)$.

(6 Marks)

Question 6

(a) A space craft has 100,000 components($n \to \infty$). The probability of any one component being defective is 2×10^{-5} ($p \to 0$). The mission will be in danger if five or more components become defective. Find the probability of such an event.

(10 Marks)

(b) A manufacturer checks for contamination on their storage disks. The mean value is 0.1 contaminants per square centimetre, with a disk surface of 100 square centimetres. What is the probability of five or more contaminants on the disks?

(10 Marks)

Question 7

Suppose that X and Y are independent random variables with the same probability density function (pdf) f (x).

(a) Write down, without proof, a formula for the pdf of X + Y.

(2 Marks)

(b) Suppose that f (x) = x/2 for $0 < x < 2$ (and f (x) = 0 elsewhere). Find the pdf of W = X + Y for $0 < w < 2$ and for $2 < w < 4$.

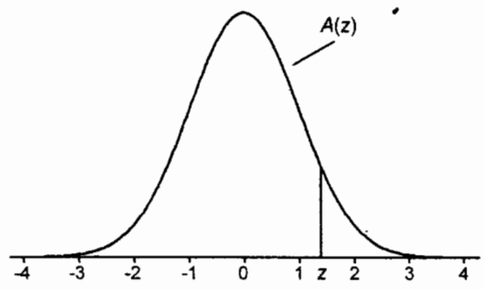
(12 Marks)

(c) Find the pdf of $V = (X - 1)^2$.

(6 Marks)

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

TABLE A.2

t Distribution: Critical Values of *t*

		Significance level					
Degrees of freedom	*Two-tailed test:* / *One-tailed test:*	10% / 5%	5% / 2.5%	2% / 1%	1% / 0.5%	0.2% / 0.1%	0.1% / 0.05%
1		6.314	12.706	31.821	63.657	318.309	636.619
2		2.920	4.303	6.965	9.925	22.327	31.599
3		2.353	3.182	4.541	5.841	10.215	12.924
4		2.132	2.776	3.747	4.604	7.173	8.610
5		2.015	2.571	3.365	4.032	5.893	6.869
6		1.943	2.447	3.143	3.707	5.208	5.959
7		1.894	2.365	2.998	3.499	4.785	5.408
8		1.860	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.250	4.297	4.781
10		1.812	2.228	2.764	3.169	4.144	4.587
11		1.796	2.201	2.718	3.106	4.025	4.437
12		1.782	2.179	2.681	3.055	3.930	4.318
13		1.771	2.160	2.650	3.012	3.852	4.221
14		1.761	2.145	2.624	2.977	3.787	4.140
15		1.753	2.131	2.602	2.947	3.733	4.073
16		1.746	2.120	2.583	2.921	3.686	4.015
17		1.740	2.110	2.567	2.898	3.646	3.965
18		1.734	2.101	2.552	2.878	3.610	3.922
19		1.729	2.093	2.539	2.861	3.579	3.883
20		1.725	2.086	2.528	2.845	3.552	3.850
21		1.721	2.080	2.518	2.831	3.527	3.819
22		1.717	2.074	2.508	2.819	3.505	3.792
23		1.714	2.069	2.500	2.807	3.485	3.768
24		1.711	2.064	2.492	2.797	3.467	3.745
25		1.708	2.060	2.485	2.787	3.450	3.725
26		1.706	2.056	2.479	2.779	3.435	3.707
27		1.703	2.052	2.473	2.771	3.421	3.690
28		1.701	2.048	2.467	2.763	3.408	3.674
29		1.699	2.045	2.462	2.756	3.396	3.659
30		1.697	2.042	2.457	2.750	3.385	3.646
32		1.694	2.037	2.449	2.738	3.365	3.622
34		1.691	2.032	2.441	2.728	3.348	3.601
36		1.688	2.028	2.434	2.719	3.333	3.582
38		1.686	2.024	2.429	2.712	3.319	3.566
40		1.684	2.021	2.423	2.704	3.307	3.551
42		1.682	2.018	2.418	2.698	3.296	3.538
44		1.680	2.015	2.414	2.692	3.286	3.526
46		1.679	2.013	2.410	2.687	3.277	3.515
48		1.677	2.011	2.407	2.682	3.269	3.505
50		1.676	2.009	2.403	2.678	3.261	3.496
60		1.671	2.000	2.390	2.660	3.232	3.460
70		1.667	1.994	2.381	2.648	3.211	3.435
80		1.664	1.990	2.374	2.639	3.195	3.416
90		1.662	1.987	2.368	2.632	3.183	3.402
100		1.660	1.984	2.364	2.626	3.174	3.390
120		1.658	1.980	2.358	2.617	3.160	3.373
150		1.655	1.976	2.351	2.609	3.145	3.357
200		1.653	1.972	2.345	2.601	3.131	3.340
300		1.650	1.968	2.339	2.592	3.118	3.323
400		1.649	1.966	2.336	2.588	3.111	3.315
500		1.648	1.965	2.334	2.586	3.107	3.310
600		1.647	1.964	2.333	2.584	3.104	3.307
∞		1.645	1.960	2.326	2.576	3.090	3.291