

**UNIVERSITY OF SWAZILAND
SUPPLEMENTARY EXAMINATION JULY 2012**

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

TITLE OF PAPER:	PROBABILITY AND STATISTICS
COURSE NUMBER:	EE301
TIME ALLOWED:	THREE HOURS

INSTRUCTIONS:

1. There are SEVEN questions in this paper. **Answer any FIVE questions.**
 2. Each question carries 20 marks.
 3. Marks for different sections are shown on the right hand margin.
 4. Statistical tables are attached at the end.
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THIS PAPER HAS FOUR (4) PAGES INCLUDING THIS PAGE

Question 1

An e-bookseller uses one of four shipping companies to send packages to its customers. Any package can be sent with one and only one of these companies. Define the following events:

- C_1 : the package is shipped with company 1
- C_2 : the package is shipped with company 2
- C_3 : the package is shipped with company 3
- C_4 : the package is shipped with company 4

The bookseller uses the shipping companies with the following probabilities:

$$P(C_1) = 0.5, P(C_2) = 0.25, P(C_3) = 0.125, P(C_4) = 0.125:$$

Let X be the event that the package arrives on time at its destination. Depending on the shipping company used, the probability of X varies:

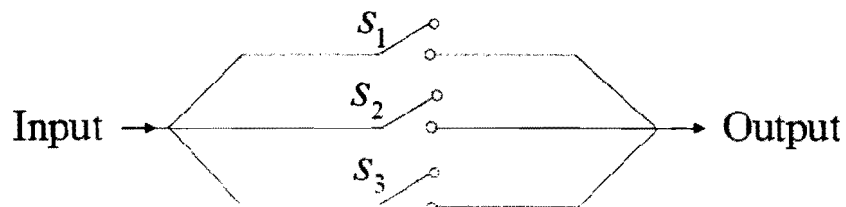
$$P(X|C_1) = 0.85, P(X|C_2) = 0.9, P(X|C_3) = 0.8, P(X|C_4) = 0.8:$$

- Compute the numerical value of $P(C_2 \cup C_3)$.
- Given that a package has arrived on time what is the probability that it was shipped with company C_1 ? In other words, compute the numerical value of $P(C_1|X)$.
- Are the events C_1 and X independent? Justify your answer.

(5 + 8 + 7 Marks)

Question 2

Three switches connected in parallel operate independently. Each switch remains closed with probability p .



- Find the probability of receiving an input signal at the output. (10 Marks)
- Find the probability that switch S_1 is open given that an input signal is received at the output. (10 Marks)

Question 3

The random variable X has the binomial distribution with probability mass function

$$P(X = x) = \binom{2}{x} p^x (1-p)^{2-x}, \quad x = 0, 1, 2; \quad 0 < p < 1.$$

- (a)
- (i) Write down $E(X)$, $\text{Var}(X)$ and $P(X = 2)$ in terms of the parameter p . (3 Marks)
 - (ii) Also find $P(X = 0 \mid X < 2)$ and $P(X = 1 \mid X < 2)$, simplifying your answers as far as possible. (4 Marks)
- (b) Let $Y = X_1 + X_2 + \dots + X_{100}$ be the sum of 100 independent random variables, each distributed as X .
- (i) Explain why Y has the $B(200, p)$ distribution. (2 Marks)
 - (ii) Use a suitable approximation to find $P(Y > 140)$ when $p = \frac{2}{3}$. (4 Marks)
 - (iii) Use a suitable approximation to find $P(Y > 2)$ when $p = 0.02$. (3 Marks)
 - (iv) Use a suitable approximation to find $P(Y \leq 197)$ when $p = 0.98$. (4 Marks)

Question 4

Flaws in lengths of fibre optic cable made by Company A occur in a Poisson process at rate λ_A per metre length, so that the number of flaws X in a length of l metres of rope has the Poisson probability mass function

$$P(X = x) = \frac{\exp(-\lambda_A l) \cdot (\lambda_A l)^x}{x!}, \quad x = 0, 1, 2, \dots; \quad \lambda_A > 0.$$

- (a) Find the probability that there are (i) no flaws, (ii) more than 2 flaws, in a 1000-metre length of rope made by company A, given that $\lambda_A = 0.002$. (4 Marks)
- (b) Company B makes similar cable, indistinguishable in appearance from that made by Company A, in which flaws occur in a Poisson process at rate $\lambda_B = 0.003$ per metre. A communications system is installed with 100 metres of rope from Company A and 100 metres of rope from Company B. Assuming that the lengths of cable supplied by A and B are independent, find the probability that (i) there are no flaws, (ii) there is exactly one flaw, in the installation. (5 Marks)
- (c) A telecommunications company buys 75% of cable from Company A and 25% from Company B. The supplier's label has become detached from a drum of cable of length 2

km which is found to have 7 flaws. Find the probability that this drum was supplied by Company A.

(6 Marks)

- (d) Suppose, instead, that the cable in this drum had been found to have 8 flaws. Find the probability that this drum was supplied by Company A. Compare this probability with your answer to part (c) and comment.

(5 Marks)

Question 5

- (a) A radioactive source of material emits a radioactive particle with probability $1/100$ in each second. Let X be the number of particles emitted in one hour.

(i) What is the distribution of X and its parameter?

(5 Marks)

(ii) Sketch the pmf of X .

(5 Marks)

- (b) An electrical component has a lifetime X that is exponentially distributed with parameter $\lambda = 1/10$ per year. What is the probability the component is still alive after 5 years?

(10 Marks)

Question 6

- (a) A space craft has 100,000 components ($n \rightarrow \infty$). The probability of any one component being defective is 2×10^{-5} ($p \rightarrow 0$). The mission will be in danger if five or more components become defective. Find the probability of such an event.

(10 Marks)

- (b) A manufacturer checks for contamination on their storage disks. The mean value is 0.1 contaminants per square centimetre, with a disk surface of 100 square centimetres. What is the probability of five or more contaminants on the disks?

(10 Marks)

Question 7

A company manufacturing light bulbs is testing a new model. The company is going to test the hypothesis that the mean life time is 1000 hours vs. the alternative hypothesis that it is less than 1000 hours at the significance level $\alpha = 0.02$. Assume that the population distribution for life time is approximately normal.

A sample of 16 light bulbs are found to have sample mean $\bar{x} = 987.5$ hours and sample variance $S^2 = 400$.

- (a) State the critical region and answer whether the null hypothesis H_0 is rejected.

(12 Marks)

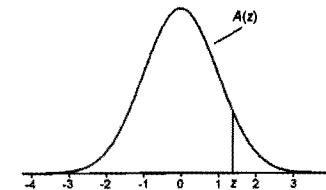
- (b) Find a 90% confidence interval for the population variance σ^2 .

(8 Marks)

STATISTICAL TABLES

- Cumulative normal distribution
- Critical values of the *t* distribution
- Critical values of the *F* distribution
- Critical values of the chi-squared distribution

TABLE A.1
Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

TABLE A.3 (continued)

F Distribution: Critical Values of F (0.1% significance level)

v_1	25	30	35	40	50	60	75	100	150	200
1	6.2605	6.2605	6.2605	6.2605	6.2605	6.2605	6.2605	6.2605	6.2605	6.2605
2	999.46	999.47	999.47	999.47	999.48	999.48	999.48	999.49	999.49	999.49
3	125.84	125.45	125.17	124.96	124.66	124.47	124.27	124.07	123.87	123.77
4	45.70	45.43	45.23	45.09	44.88	44.75	44.61	44.47	44.33	44.26
5	25.08	24.87	24.72	24.60	24.44	24.33	24.22	24.12	24.01	23.95
6	16.85	16.67	16.54	16.44	16.31	16.21	16.12	16.03	15.93	15.89
7	12.69	12.53	12.41	12.33	12.20	12.12	12.04	11.95	11.87	11.82
8	10.26	10.11	10.00	9.92	9.80	9.73	9.65	9.57	9.49	9.45
9	8.69	8.55	8.46	8.37	8.26	8.19	8.11	8.04	7.96	7.93
10	7.60	7.47	7.37	7.30	7.19	7.12	7.05	6.98	6.91	6.87
11	6.81	6.68	6.59	6.52	6.42	6.35	6.28	6.21	6.14	6.10
12	6.22	6.09	6.00	5.93	5.83	5.76	5.70	5.63	5.56	5.52
13	5.75	5.63	5.54	5.47	5.37	5.30	5.24	5.17	5.10	5.07
14	5.38	5.25	5.17	5.10	5.00	4.94	4.87	4.81	4.74	4.71
15	5.07	4.95	4.86	4.80	4.70	4.64	4.57	4.51	4.44	4.41
16	4.82	4.70	4.61	4.54	4.45	4.39	4.32	4.26	4.19	4.16
17	4.60	4.48	4.40	4.33	4.24	4.18	4.11	4.05	3.98	3.95
18	4.42	4.30	4.22	4.15	4.06	4.00	3.93	3.87	3.80	3.77
19	4.26	4.14	4.06	3.99	3.90	3.84	3.78	3.71	3.65	3.61
20	4.12	4.00	3.92	3.86	3.77	3.70	3.64	3.58	3.51	3.48
21	4.00	3.88	3.80	3.74	3.64	3.58	3.52	3.46	3.39	3.36
22	3.89	3.78	3.70	3.63	3.54	3.48	3.41	3.35	3.28	3.25
23	3.79	3.68	3.60	3.53	3.44	3.38	3.32	3.25	3.19	3.16
24	3.71	3.59	3.51	3.45	3.36	3.29	3.23	3.17	3.10	3.07
25	3.63	3.52	3.43	3.37	3.28	3.22	3.15	3.09	3.03	2.99
26	3.56	3.44	3.36	3.30	3.21	3.15	3.08	3.02	2.95	2.92
27	3.49	3.38	3.30	3.23	3.14	3.08	3.02	2.96	2.89	2.86
28	3.43	3.32	3.24	3.18	3.09	3.02	2.96	2.90	2.83	2.80
29	3.38	3.27	3.18	3.12	3.03	2.97	2.91	2.84	2.78	2.74
30	3.33	3.22	3.13	3.07	2.98	2.92	2.86	2.79	2.73	2.69
35	3.13	3.02	2.93	2.87	2.78	2.72	2.66	2.59	2.52	2.49
40	2.98	2.87	2.79	2.73	2.64	2.57	2.51	2.44	2.38	2.34
50	2.79	2.68	2.60	2.53	2.44	2.38	2.31	2.25	2.18	2.14
60	2.67	2.55	2.47	2.41	2.32	2.25	2.19	2.12	2.05	2.01
70	2.58	2.47	2.39	2.32	2.23	2.16	2.10	2.03	1.95	1.92
80	2.52	2.41	2.32	2.26	2.16	2.10	2.03	1.96	1.89	1.85
90	2.47	2.36	2.27	2.21	2.11	2.05	1.98	1.91	1.83	1.79
100	2.43	2.32	2.24	2.17	2.08	2.01	1.94	1.87	1.79	1.75
120	2.37	2.26	2.18	2.11	2.02	1.95	1.88	1.81	1.73	1.68
150	2.32	2.21	2.12	2.06	1.96	1.89	1.82	1.74	1.66	1.62
200	2.26	2.15	2.07	2.00	1.90	1.83	1.76	1.68	1.60	1.55
250	2.23	2.12	2.03	1.97	1.87	1.80	1.72	1.65	1.56	1.51
300	2.21	2.10	2.01	1.94	1.85	1.78	1.70	1.62	1.53	1.48
400	2.18	2.07	1.98	1.92	1.82	1.75	1.67	1.59	1.50	1.45
500	2.17	2.05	1.97	1.90	1.80	1.73	1.65	1.57	1.48	1.43
600	2.16	2.04	1.96	1.89	1.79	1.72	1.64	1.56	1.46	1.41
750	2.15	2.03	1.95	1.88	1.78	1.71	1.63	1.55	1.45	1.40
1000	2.14	2.02	1.94	1.87	1.77	1.69	1.62	1.53	1.44	1.38

TABLE A.4

χ^2 (Chi-Squared) Distribution: Critical Values of χ^2

Degrees of freedom	Significance level		
	5%	1%	0.1%
1	3.841	6.635	10.828
2	5.991	9.210	13.816
3	7.815	11.345	16.266
4	9.488	13.277	18.467
5	11.070	15.086	20.515
6	12.592	16.812	22.458
7	14.067	18.475	24.322
8	15.507	20.090	26.124
9	16.919	21.666	27.877
10	18.307	23.209	29.588