## UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE<br>DEPARTMENT OF ELECTRICAL \& ELECTRONIC ENGINEERING DIGITAL SYSTEMS I<br>COURSE CODE - EE322<br>MAIN EXAMINATION DECEMBER 2011<br>DURATION OF THE EXAMINATION - 3 HOURS

## INSTRUCTIONS TO CANDIDATES

i. There are FIVE questions in this paper. Answer any FOUR questions only.
ii. Each question carries equal marks.
iii. Show all your steps clearly in any calculations.
iv. State clearly any assumptions made.
v. Start each new question on a fresh page.

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## Question 1

(a) Using the tabular method, minimize the following Boolean function:

$$
\begin{equation*}
F(A, B, C, D)=\sum(0,2,3,5,7,8,10,11,14,15) \tag{10}
\end{equation*}
$$

(b) Simplify the following using a k-map:
(i) $\quad \mathrm{F}=\mathrm{C}^{\prime} \mathrm{D}+\mathrm{ABC} C^{\prime}+\mathrm{ABD}^{\prime}+\mathrm{A}^{\prime} \mathrm{B}^{\prime} \mathrm{D}$
(ii) $\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D})=\Sigma(0,6,8,13,14)$
(c) Using Boolean algebra simplify the following Boolean expression to a minimum number of literals:

$$
\begin{equation*}
F=w x^{\prime}+y^{\prime} z^{\prime}+w^{\prime} y z^{\prime} \tag{4}
\end{equation*}
$$

## Question 2

Analyse the combinational circuit shown in Figure Q2 and determine its truth table. Show all working.


Figure Q2: Diagram for question 2

## Question 3

a) Derive the Boolean expressions for the output sum and output carry in a full adder circuit with inputs $x_{i}, y_{i}$, and $c_{i}$.
b) Assuming that the carry propagate and carry generate are defined as

$$
\begin{aligned}
& P_{i}=\mathrm{x}_{i}+\mathrm{y}_{i} \\
& G_{i}=\mathrm{x}_{i} \mathrm{y}_{i}
\end{aligned}
$$

respectively, show that the output carry and output sum of a full adder becomes

$$
\begin{align*}
& \mathrm{C}_{i+1}=\left(\mathrm{C}_{i}^{\prime} \mathrm{G}_{i}^{\prime}+\mathrm{P}_{i}^{\prime}\right)^{\prime}  \tag{8}\\
& \mathrm{S}_{i}=\left(\mathrm{P}_{i} \mathrm{G}_{i}^{\prime}\right) \oplus \mathrm{C}_{i} \tag{7}
\end{align*}
$$

c) Design a half-subtractor circuit with inputs x and y and outputs D and B . The circuit subtracts the bits $x-y$ and places the difference in $D$ and the borrow in B .

## Question 4

(a) Figure Q 4 is a waveform showing the behavior of a combinational circuit. $\mathrm{A}, \mathrm{B}$, and $C$ are inputs, and $X$ and $Y$ are outputs. Implement the circuit and determine the function the circuit performs.
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Figure Q4: Diagram for question 4 (a)
(b) Design a 4-bit Magnitude Comparator which compares two 4-bit binary numbers $\mathrm{A}, \mathrm{B}$ and determines whether $\mathrm{A}<\mathrm{B}, \mathrm{A}>\mathrm{B}$, or $\mathrm{A}=\mathrm{B}$.

## Question 5

a) A combinational circuit is defined by the following three Boolean functions:

$$
\begin{aligned}
& \mathrm{F}_{1}(\mathrm{~A}, \mathrm{~B}, \mathrm{C})=\sum(2,4,7) \\
& \mathrm{F}_{2}(\mathrm{~A}, \mathrm{~B}, \mathrm{C})=\sum(0,3) \\
& \mathrm{F}_{3}(\mathrm{~A}, \mathrm{~B}, \mathrm{C})=\sum(0,2,3,4,7)
\end{aligned}
$$

Design the circuit with a decoder and a minimum number of external NAND gates.
b)
i. Specify the truth table of an hexadecimal-to-binary priority encoder. In your truth table, provide an output V to indicate that at least one of the inputs is present. The input with the highest subscript number has the highest priority.
ii. What will be the value of the four outputs if inputs D9 and D6 are at 1 at the same time?
c) Implement the following Boolean function with a $4 \times 1$ multiplexer and external gates.

$$
\mathrm{F}(\mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D})=\Sigma(0,1,3,4,8,9,15)
$$

d) An $8 \times 1$ multiplexer has inputs $\mathrm{A}, \mathrm{B}$, and C connected to the selection inputs $S_{2}, S_{1}$, and $S_{0}$, respectively. The data inputs $I_{0}$ through $I_{7}$ are as follows: $\mathrm{I}_{1}=\mathrm{I}_{2}=\mathrm{I}_{7}=0 ; \mathrm{I}_{3}=\mathrm{I}_{5}=1 ; \mathrm{I}_{0}=\mathrm{I}_{4}=\mathrm{D}$; and $\mathrm{I}_{6}=\mathrm{D}^{\prime}$. Determine the Boolean function that the multiplexer implements (express it is sum of products).

