UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRICAL & ELECTRONIC ENGINEERING

MAIN EXAMINATION, DECEMBER 2011

TITLE OF PAPER: SIGNALS & SYSTEMS I

COURSE CODE : EE331

TIME ALLOWED: THREE (3) HOURS

INSTRUCTIONS : ***ANSWER QUESTION 1 AND ANY OTHER THREE QUESTIONS***

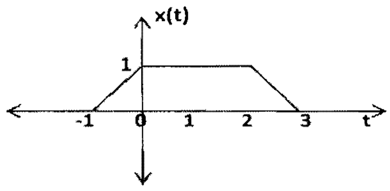
EACH QUESTION CARRIES 25 MARKS

MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT HAND MARGIN

DO NOT OPEN THE PAPER UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR

QUESTION 1

(a) With the help of the following signal, *x(t)*, discuss the three different types of transformations of the independent variable, *t*. [8]



(b) From the definition of the average power of a periodic signal, (i) derive Parseval's theorem and (ii) explain its spectral interpretation. [4]

(c) Discuss the two classifications of signals that have to do with the measure of the size of a signal. Give examples. [4]

(d) With the help of an example, explain the *sifting property* of the unit impulse signal [2]

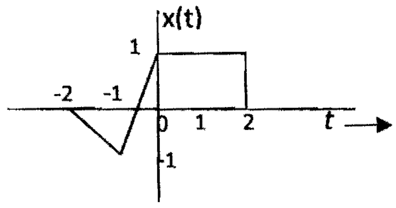
(e) Discuss any two classifications or properties of systems, giving an example for each. [4]

(f) What is the main idea behind Fourier series? [1]

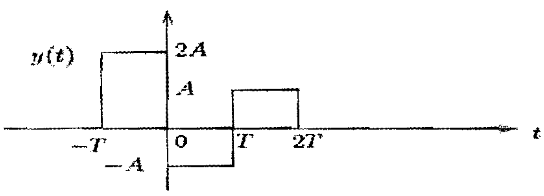
(g) Prove the linearity property of the Laplace Transform [2]

QUESTION 2

(a)(i) Give and explain two possible sequences of operation by which the signal transformation $y(t) = x(2t - 6)$ can be realized from x(t). (ii) Sketch and label y(t) from the given x(t) below. [8]



(b) For the waveform y(t) shown below:



Determine the energy and the mean value. [4]

(c)(i) Sketch the continuous-time signal $a(t) = A \times rect\left(\frac{t-t_0}{T}\right)$ and (ii) derive an expression for its energy. [4]

(d) Sketch the even and odd components of the signal $x(t) = 2rect\left(\frac{t}{3} - 2\right)$ [4]

(e) Determine the fundamental period of the discrete-time signal $x[n] = \cos\left(\frac{\pi}{3}n\right) + \sin\left(\frac{\pi}{4}n\right)$. [2]

(f) Show that the following two signals are orthogonal

$rect\left(\frac{t}{2}\right)$ and $t \times rect\left(\frac{t}{2}\right)$ [3]

QUESTION 3

(a) Derive the exponential Fourier series from the trigonometric Fourier series. [5]

(b)(i) Find the Fourier coefficients and (ii) sketch the corresponding line spectra for the following signal

$x(t) = 1 + sin\omega_0 t + 2cos\omega_0 t + \cos\left(2\omega_0 t + {}^{\pi}/_{4}\right)$ [10]

(c) Obtain the trigonometric Fourier series of a periodic voltage signal defined by the following equation. [10]

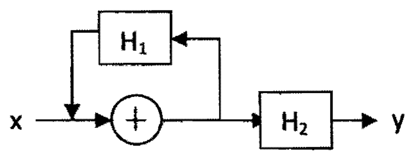
$$x(t) = x(t+2) = \begin{cases} 1, & -1 \le t \le 0 \\ 2, & 0 \le t \le 1 \end{cases}$$

QUESTION 4

(a) Show that the system described below is a time-varying-parameter system [2]

$$y(t) = (\sin t)x(t-2)$$

(b) Determine the input-output relationship for the system shown below. [4]



(c) Determine if the following systems are: (i) time-invariant, (ii) memoryless, (iii) Linear, or (iv) causal. Justify your answers.

1. $y(t) = \frac{x(t)}{2+x(t-5)}$ [5]

2. $y[n] = x[3-n]$ [5]

(d) Differentiate between a stable system and an unstable system, giving any suitable examples. [4]

(e) Show that a system with the input x(t) and output y(t) related by the following equation is invertible. [3]

$$y(t) = x(t+2)$$

(f) Sketch a block diagram representation of the system defined by the following difference equation. [2]

$$y[n] + \frac{3}{5}y[n-1] = 5x[n]$$

QUESTION 5

(a) Find the Laplace transform and the region of convergence (ROC) of the following signal, *x(t)*. Sketch ROC. [6]

$$\frac{-2}{5}e^{-3t}u(-t)$$

(b) Compute the impulse response of a system with the transfer function, $H(s)$, as given below. Assume the input $x(t)$ and output $y(t)$ are related by $Y(s) = H(s)X(s)$ [5]

$$H(s) = \frac{2s^2 + 5}{s^2 + 3s + 2}$$

(c) Use Parseval's theorem to calculate the average power of the following signal. [7]

$$x(t) = 7 - 10\cos\left(40\pi t - {}^{\pi}/_{3}\right) + 4\sin(120\pi)$$

(d)(i) What is aliasing? A CD system has a sample rate of 44 KHz, (ii) what is the highest frequency that can be sampled by this system without aliasing? [2]

(e) A causal digital processor with input response *h[n]* is fed with input sequence *x[n]*, as given below. Determine the output y[n]. [5]

$$h[n] = [3, 4, 2, 3]$$
$$x[n] = 2\delta[n] + \delta[n-1] + 3\delta[n-2] + 5\delta[n-4]$$