# UNIVERSITY OF SWAZILAND 

## FACULTY OF SCIENCE <br> Department of Electronic and Electrical Engineering

## MAIN EXAMINATION 2011

Title of the Paper:<br>Electromagnetic Fields I

Course Number: EE341
Time Allowed: Three Hours.

Instructions:

1. To answer, pick any to sum a total of $100 \%$ from

14 questions in the following pages.
2. Each question carriers 10 points.
3. The answer is better written in the space provided in the question book. Use the answer book as a scratch pad.
4. This paper has 9 pages, including this page.

DO NOT OPEN THE PAPER
UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Q1: Given a scalar function $f(x, y, z)=x \cdot y$, find (i) $\int f \cdot d \vec{l}$ and (ii) $\int f \cdot d l$ along a straight line from $(0,0,0)$ to $(1,1,0)$. $\quad 10 \mathrm{pts}$ ( 5 pts for each (i) and (ii))

Q2: Given a scalar function, $h(x, y)=\left(x^{2}+y^{2}\right) / 2 x$, the height of a slanted cone shown in Fig. Q2-1, (i) calculate graphically the maximum change (gradient) of the height at the location $P_{x}(6,6)$ and the direction of the change; (ii) calculate the same but analytically. Check if the two answers are close. $\quad \mathbf{1 0} \mathrm{pts}(5 \mathrm{pts}$ for each (i) and (ii), 3 pts for the direction part)


Q3: Given two field patterns shown in Fig. Q3-1 and -2, by inspection determine and mark the area which has curl $\neq 0$ or $\operatorname{div} \neq 0$ or both $\neq 0$ of the pattern. Then analytically calculate the non-zero curl or divergence to prove. Take closed surface anywhere in the pattern but must be specified. The fields are in xy-plane only, no contribution in z-axis top and bottom. The closed surface may be cubically or cylindrically bounded. $\quad 10 \mathrm{pts}$ ( 5 pts for each pattern.)
(b) $\mathbf{A}=-\hat{\mathbf{x}} \sin 2 y+\hat{\mathbf{y}} \cos 2 x$, for $-\pi \leq x, y \leq \pi$


Fig. Q3-1
(d) $\mathrm{A}=-\hat{\mathbf{x}} \cos x+\hat{\mathbf{y}} \sin y$, for $-\pi \leq x, y \leq \pi$


Fig. Q3-2

Q4: List any five pairs of dual equation in electromagnetic fields. $\mathbf{1 0} \mathbf{~ p t s}$ ( 2 pts for each pair)

| term | Electric Fields | Magnetic Fields |
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Q5: A coaxial cable has a inner radius $r_{i}$ and outer radius $r_{0}$ with insulation material $\varepsilon / \mu_{0}$. Consider no end fringing effects. (i) Find the total electric energy stored in this 1 meter long cable, energized by a source charge $q_{1} \mathrm{Coul} / \mathrm{Mtr}$. (ii) Find the total magnetic energy stored in this 1 meter long cable, energized by a source current Is. 10 pts ( 5 pts for each)

Q6:An infinitely long line charge with a line density $+\mathrm{q}_{1} \mathrm{Coul} / \mathrm{Mtr}$ is located d Mtr above an infinitive perfect conducting plane. Find the charge density on the plane. Use the image method. Is there any dual method in static magnetic fields and give the reason behind? $\quad \mathbf{1 0}$ pts ( 6 pts for the first question, 4 pts for the second).


Fig. Q6-1

Q7: A coaxial cable has a inner radius $r_{i}$ and outer radius $r_{0}$ with insulation material $\varepsilon / \mu_{0}$. Consider no end fringing effects. (i) Calculate the cable per unit inductance and capacitance. (ii) the Characteristic impedance $z_{0}$. $\quad 10 \mathrm{pts}$ ( 4 pts for each answer in (i) and 2 pts for (ii))

Q8: An electric dipole antenna has a dipole moment $1 / 9$ coul-mtr and its direction is oriented in the z-axis. Calculate (i) the electric field at 1 KM away with $\theta=0^{\circ}$ and (ii) the same with $\theta=90^{\circ}$. (iii) Comment on the direction of the two fields with respect to the dipole orientation. 10 pts ( 4 pts for (iii), 3 pts for each of (i) and (ii).)


Fig. Q8-1

Q9: Two infinitely long line charges in the direction of $z$-axis, one carries a charge, $+\mathrm{q} \mathrm{coul} / \mathrm{mtr}$ located at the center of the cylindrical coordinates (possibly a Cartesian) and the other carries -0.2 q coul $/ \mathrm{mtr}$ at d meters away from z-axis. Find the zero potential surface. 10 pts

Q10: Prove (i) which equation or law in electric fields will degenerate into Kirchhoff's Voltage Law, specifying the necessary conditions; (ii) which will degenerate into Kirchhoff's Current Law likewise. 10 pts (5 pts for each)

Q11: Two parallel and infinitively long conductors are separated 2d mMtr apart. Set the conductors along $z$-axis lying on $x z$-plane and the coordinates center at middle point of the two conductors. The conductors carry a current of I Amps in opposite direction and so each is stored an opposite charge of $q \mathrm{coul} / \mathrm{mtr}$ on the line. (i) Determine the B-field away from y -axis ( $0, \infty$ ); (ii) determine the E field likewise. Notice any special point about these two fields. 10 pts ( 4 pts each, 2 pts for the notice)(hint: application of ready known formula is recommendded)

Q12(i) Show that if no surface current densities exist at the parallel interfaces shown in Fig. Q12-1, the relationship between $\theta_{4}$ and $\theta_{1}$ is independent of $\mu_{2}$. (ii) Show the same for independent of $\varepsilon_{2}$ for electric fields if no surface charge densities exist likewise. $\quad 10$ pts ( 5 pts for each)


Q13: A current coil of radius $r_{o}$ carries a current I. Determine the vector potential of this coil at the point on its axis and $z$ meters away from the coil plane. 10pts


Q14: A series magnetic circuit with a uniform thickness of 6 cm is shown in Fig. Q14-1 with all dimensions in centimeters. If the current through the 500 -turn coil is 1.0 A , (i) determine the current in the 600 -turn coil in order to maintain flux of 1.0 mWb in the air gap. Assume the permeability of the magnetic material is $\mu_{\mathrm{r}}=500$. (ii) Compute the ratio of the mmf drop across the air gap to the applied mmf. $\mathbf{1 0} \mathbf{~ p t s}$ ( 8 pts for (i), 2 pts for (ii))


Fig. Q14-1

