# UNIVERSITY OF SWAZILAND <br> FACULTY OF SCIENCE <br> Department of Electrical and Electronic Engineering 

# SUPPLEMENTARY EXAMINATION JULY 2012 

## Title of Paper: <br> Electromagnetic Fields I

Course Code: EE341<br>Time: THREE Hours

## Instructions

1. To answer, in the following pages, pick any questions to sum a total of $100 \%$. Beware that it is your responsibility to mark the table below indicating the questions you have picked.

| $(1)$ | $(2)$ | $(3)$ | $(4)$ | $(5)$ | $(6)$ | $(7)$ | $(8)$ | $(9)$ | $(10)$ | $(11)$ | $(12)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $15 \%$ | $10 \%$ | $15 \%$ | $10 \%$ | $15 \%$ | $10 \%$ | $10 \%$ | $15 \%$ | $15 \%$ | $10 \%$ | $10 \%$ | $15 \%$ |

2. Each question carries 10 or 15 points as indicated in the table above.
3. The answer is better written in the space provide in this question paper. Use the answer book as a scrap pad.
4. This paper has 9 pages, including this page.

DO NOT OPEN THE PAPER
UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Q1: Given a scalar function $f(x, y, z)=x^{2}+y$, find (i) $\int f \cdot d \vec{l}$ and (ii) $\int f \cdot d l$ along a straight line $y=x+1$ from $x=0$ to $x=1 . \quad 15$ pts ( 8 pts for (i) and 7 for (ii))

Q2: Given a scalar function, $h(x, y, z)=\left(x^{2} \cdot y+z\right)$ calculate: (i) the gradient of $h(x, y, z)$, (ii) the direction of the gradient. $\mathbf{1 0} \mathbf{p t s}$ ( 5 pts for each)

Q3: Given the field pattern equation below, analyze and locate (i) the position of non-zero and zero curl, (ii) do the same for divergence. The fields are in $x y$-plane only, no contribution in $z$-axis top and bottom. 15 pts (4pts for Curl, 2pts for location; 4pts for Div, 3 for location $\operatorname{Div}=0$ and 2 for $\operatorname{Div} \neq 0$ )

$$
\vec{f}=\vec{u}_{x} \sin x \cdot \cos y+\bar{u}_{y} \sin y \cdot \cos x
$$

Q4: A parallel plate cable, shown in Fig. Q4-1, has a width " w " and separation " d " with insulation material $\varepsilon / \mu_{0}$. Consider no fields outside of the space between the plates (that is no end fringing effects). (i) Find the total electric energy stored in this 1 meter long cable, energized by a
 source charge $q_{1}$ Coul/Mtr. (ii) Find the total magnetic energy stored in this 1 meter long cable, energized by a source current $I s . \quad 10 \mathrm{pts}$ ( 5 pts for (i), 5 for (ii))

Q5: The same cable as in Q4, Calculate the cable (i) inductance and (ii) capacitance per unit length. (iii) the Characteristic impedance $z_{0}$. 15 pts (5 pts for each answer)

Q6:A point charge of $+q$ Coul is located $d$ Mtr above an infinitively large perfect conducting plane. Find the charge density on the plane. Use the image method. That is to find the $\vec{D}$ on the conducting plane. 10 pts .


Fig. Q6-1

Q7: List five pairs of dual equations $\mathbf{1 0} \mathbf{p t s}$ ( 2 pts for each pair)

| term | Time-domain | Phasor-domain |
| :--- | :--- | :--- |
|  | $A \cos (\omega t+45)$ |  |
|  | $\frac{d}{d t} A \cos (\omega t+\varphi)$ |  |
|  | Electric | Magnetic |
|  | $\vec{D}=\varepsilon \vec{E}$ |  |
|  |  | $\vec{F}=I d \vec{l} \times \vec{B}$ |
|  | $\vec{\nabla} \circ \vec{D}=q_{r}$ |  |

Q8: A magnetic dipole antenna has a dipole moment " $m$ " amp-mtr ${ }^{2}$ and its direction is oriented in the $z$-axis. Calculate (i) the magnetic field at a far away distance with $\theta=0^{\circ}$ and $\theta=90^{\circ}$, (ii) What is the geometric shape of the magnetic dipole. (hint: use the dual process to get the answer from the electric dipole shown on the right) $\quad 15$ pts (8 pts for (i), 7 pts for (ii).)


Fig. Q8-1

Q9: Two parallel and infinitively long conductors are separated 2 dmMtr apart. Set the conductors along z -axis lying on $x z$-plane and the coordinates center at middle point of the two conductors. The conductors carry a current of I Amps in opposite direction and so each is stored an opposite charge of $q$ coul/mtr on the line. (i) Determine the B -field away from y-axis


Fig. Q9-1 $(0, \infty)$; (ii) determine the E field likewise. (iii) Notice any special point about these two fields. (hint: application of ampere's law or Gauss law is recommended). $\quad \mathbf{1 5} \mathrm{pts}$ ( 5 pts for each)

Q10: Find $\alpha_{4}$ if no surface charge densities exist at the two parallel interfaces shown in Fig. Q10-1. In fact, $\alpha_{1}=0, \alpha_{3}=\alpha_{2}+15 . \quad 10$ pts ( 5 pts for $\alpha_{2}, 5 \mathrm{pts}$ for $\alpha_{4}$ )


Q11: A current coil of radius $r_{0}$, shown in Fig. Q11-1, carries a current I. Determine the vector potential of this coil at the point on its axis and z meters away from the coil plane. 10pts


Q12: A densely wound toroidal coil of total N turns, with an inner radius 5 cMeters and a square cross-section of 5 cMtrs on the sides, has an air gap of 0.5 cM trs wide shown in Fig. Q12-1. If the current through the coil is 1.0 A , (i) determine the coil turns N in order to maintain a flux of 1.0 mWb in the air gap. Assume the permeability of the magnetic material is $\mu_{\mathrm{r}}=500$. (ii) Compute the ratio of the mmf drop across the air gap to the applied mmf. Use average radius 7.5 cM to calculate the length of the H -field. $\quad 15$ pts ( 10 pts for (i), 5 pts for (ii))


Fig. Q12-1

